1. Solve the initial value problems $\dfrac{dy}{dx} = 4x\sqrt{y-1}, \quad y(2) = 1$ and $\dfrac{dy}{dx} = y^2 \sin x, \quad y(0) = 1.$
   Can you be certain if either of these initial value problems has a unique solution?

2. Find the general solution of each of the following differential equations. (Your solutions should be real-valued.)

   (a) $y' + 4y = e^{-x}$ (c) $xy' - y = 4xy^2$ Hint: make the substitution $u = y^{-1}$

   (b) $x + xy^2 + e^{x^2} y \dfrac{dy}{dx} = 0$ (d) $y'' + 10y' + 25y = 0$ (e) $y'' - 2y' + 5y = 0$

3. (a) Solve the initial value problem $y'' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 6.$
   (b) Find the general solution of each of the following non-homogeneous differential equations.
   (i) $y'' - 4y = 8e^{2x}$ (ii) $y'' - 4y = 5\sin x$ (iii) $y'' - 4y = 4e^{2x} - 5\sin x$
   (c) Explain how the results of parts (i) and (ii) above can be used to obtain the answer to part (iii).

4. The function $y_1(x) = x$ is solution of the differential equation $x^2y'' - 2xy' + 2y = 0, \quad x > 0.$
   Use the method of reduction of order to find a second linearly independent solution $y_2(x)$ on the interval $I = (0, \infty)$. Explain why the solutions $y_1(x)$ and $y_2(x)$ are linearly independent on I, and find the general solution of this differential equation.

5. Newton's law of cooling states that the rate of change of the temperature of an object with respect to time t is proportional to difference between the temperature, $W(t)$, of the object at time t and the temperature, R, of the surrounding medium.
   Suppose that the temperature of the water in my cup is 45°F at the beginning of class, and is 50°F ten minutes later. Assume that room temperature $R = 72$°F is constant. To determine the temperature of my water at the end of class using Newton's law of cooling, you would have to solve a certain initial value problem. Write this initial value problem down. Then solve it.

6. Boyle-Mariotte's law for ideal gases. For a gas at low pressure p (and constant temperature), the rate of change of the volume $V = V(p)$ is equal to $-V/p$. Determine the volume as a function of p.