MATH 2085-1 EXAM 1 INFORMATION FALL 2005

In the wake of our most recent natural disaster, Exam 1 will take place on Monday, October 3. It will cover the material from Chapters 1 and 2 we've discussed in class (primarily §1.1–1.6 and §2.1–2.3).

Books, notes, calculators, etc. may **not** be used on the exam.

We can spend some time in class this week reviewing as necessary. You may also make use of my office hours. There may be relevant free tutoring available in 39 Allen Hall, but I haven't heard anything definitive about this service for this semester.

The focus of the above sections, and this exam, is on systems of linear equations, matrices, determinants, etc. By now, you should be comfortable:

(1) finding all solutions of a given system of linear equations;
(2) working with matrices and various matrix operations;
(3) determining if a given square matrix is invertible, and finding the inverse (using Gauss-Jordan elimination) if it is;
(4) computing the determinant of a given square matrix using cofactor expansion, elementary row operations, or a combination of the two.

I do not anticipate asking you to give formal proofs on the exam. However, this exam will have conceptual components. For instance, you should be prepared to discuss the implications of a given computation (e.g., if $\det(A) = 0$, what can you conclude?).

A few review problems are included below. This is **not** a comprehensive list. Additional problems may be found in the Exercises of the sections we've covered, the WeBWorK assignments, and the Supplementary Exercises at the ends of Chapters 1 and 2. The *Discussion & Discovery* problems at the ends of the sections also provide useful materials for reviewing conceptual aspects of this subject matter.

MATH 2085-1 EXAM 1 REVIEW PROBLEMS FALL 2005

(1) Consider the system of linear equations
$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & + & x_4 & = & 6 \\ 2x_1 & - & x_2 & + & 3x_3 & - & x_4 & = & 3 \\ -x_1 & + & x_2 & - & 2x_3 & & & = & -3 \end{array}$$
Write this system in matrix form, find a reduced row echelon matrix that is row equivalent to the coefficient matrix of this system, and find the solution set of the system.

(2) Consider the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ and the constant vector $\vec{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

(a) Use Gauss-Jordan elimination to find the inverse of A.
(b) Use your answer from part (a) to solve the system of equations $A\vec{x} = \vec{b}$.
(c) Is the transpose of A invertible? Explain.

(3) Consider the matrix $A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 3 \\ 0 & 1 & -1 & 0 \\ 1 & 3 & -2 & 4 \end{bmatrix}$.

(a) Compute the determinant of A. Does the matrix A have an inverse? Explain.
(b) If B is a 4×4 matrix with $\det(B) = -7$, what is the determinant of the product AB? Explain.
(c) Can a system $A\vec{x} = \vec{b}$ involving this matrix A, and any constant vector $\vec{b}$, be inconsistent? Explain.

(4) Consider the homogeneous system of linear equations
$$\begin{array}{rcrcrcr} (1-\lambda)x_1 & + & 2x_2 & + & 6x_3 & = & 0 \\ & & (2-\lambda)x_2 & + & 3x_3 & = & 0 \\ & & x_2 & + & (4-\lambda)x_3 & = & 0 \end{array}$$

(a) Find all values of λ for which this system has infinitely many solutions.
(b) Find the solution set of this homogeneous system when $\lambda = 1$.

(1) Consider the system of linear equations
$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & + & x_4 & = & 6 \\ 2x_1 & - & x_2 & + & 3x_3 & - & x_4 & = & 3 \\ -x_1 & + & x_2 & - & 2x_3 & & & = & -3 \end{array}$$
Write this system in vector form, find a reduced row echelon matrix that is row equivalent to the coefficient matrix of this system, and find the solution set of the system.

> In matrix form, $A\vec{x} = \vec{b}$, the system is $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & -1 & 3 & -1 \\ -1 & 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix}$.
>
> The coefficient matrix A is row equivalent to $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
>
> The solution set of the system is $\{(-s+t, s+t-3, s, t) : s, t \text{ real}\}$.

(2) Consider the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ and the constant vector $\vec{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

(a) Use Gauss-Jordan elimination to find the inverse of A.
(b) Use your answer from part (a) to solve the system of equations $A\vec{x} = \vec{b}$.
(c) Is the transpose of A invertible? Explain.

> (a) $A^{-1} = \begin{bmatrix} 5 & -1 & -2 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ (b) $\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 12 \\ 5 \\ -2 \end{bmatrix}$ (c) Yes, $(A^T)^{-1} = (A^{-1})^T$. Can you prove this?

(3) Consider the matrix $A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 3 \\ 0 & 1 & -1 & 0 \\ 1 & 3 & -2 & 4 \end{bmatrix}$.

(a) Compute the determinant of A. Does the matrix A have an inverse? Explain.
(b) If B is a 4×4 matrix with $\det(B) = -7$, what is the determinant of the product AB? Explain.
(c) Can a system $A\vec{x} = \vec{b}$ involving this matrix A, and any constant vector $\vec{b}$, be inconsistent? Explain.

> (a) $\det A = -7$. Since $\det A \neq 0$, A is non-singular.
> (b) $\det AB = \det A \det B = 49$.
> (c) Since A is non-singular, any system of equations with coefficient matrix A has a unique solution (so is consistent).

(4) Consider the homogeneous system of linear equations
$$\begin{array}{rcrcrcr} (1-\lambda)x_1 & + & 2x_2 & + & 6x_3 & = & 0 \\ & & (2-\lambda)x_2 & + & 3x_3 & = & 0 \\ & & x_2 & + & (4-\lambda)x_3 & = & 0 \end{array}$$

(a) Find all values of λ for which this system has infinitely many solutions.
(b) Find the solution set of this homogeneous system when $\lambda = 1$.

> (a) The system has infinitely many solutions if $\lambda = 1$ or $\lambda = 5$.
> (b) When $\lambda = 1$, the solution set is $\{(s, -3t, t) : s, t \text{ real}\}$.