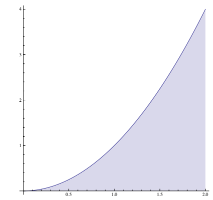
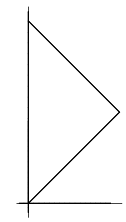
1. [14 points] Consider the iterated integral $\displaystyle\int_0^2 \int_0^{x^2} \frac{1}{x^3+1}\,dy\,dx.$
   (a) Evaluate this iterated integral.
   (b) This iterated integral is equal to a double integral: $\displaystyle\int_0^2 \int_0^{x^2} \frac{1}{x^3+1}\,dy\,dx = \iint_{\mathcal{D}} \frac{1}{x^3+1}\,dA$ where $\mathcal{D}$ is a certain region in the xy-plane. Sketch the region $\mathcal{D}$.

$$\int_0^2 \int_0^{x^2} \frac{1}{x^3+1}\,dy\,dx = \int_0^2 \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\ln(x^3+1)\Big|_0^2 = \frac{1}{3}\ln(9)$$



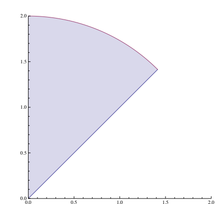
2. [14 points] Find the volume of the solid $\mathcal{W}$ bounded above by the surface $z = 2x^2y$, and bounded below by the triangular region $\mathcal{D}$ in the xy-plane with vertices $(0,0)$, $(1,1)$, and $(0,2)$.

$$\text{volume}(\mathcal{W}) = \int_0^1 \int_x^{2-x} 2x^2y\,dy\,dx = \frac{1}{3}$$



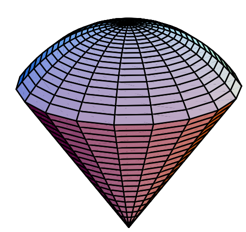
3. [14 points] Let $\mathcal{D}$ be the region in the first quadrant bounded by the vertical line $x = 0$, the line $y = x$ and the circle $x^2+y^2 = 4$. Sketch the region $\mathcal{D}$, and use polar coordinates to evaluate the double integral $\displaystyle\iint_{\mathcal{D}} x\,dA$.

$$\iint_{\mathcal{D}} x\,dA = \int_{\pi/4}^{\pi/2} \int_0^2 r\cos\theta\, r\,dr\,d\theta = \int_{\pi/4}^{\pi/2} \int_0^2 r^2\cos\theta\,dr\,d\theta = \frac{8}{3}\left(1 - \frac{\sqrt{2}}{2}\right)$$



4. [18 points] Let $\mathcal{W}$ be the solid that lies above the cone $z = \sqrt{x^2+y^2}$, and lies below the sphere $x^2+y^2+z^2 = 8$, and consider the triple integral $\displaystyle\iiint_{\mathcal{W}} (2x+3y+94z)\,dV$.
   (a) Set up an iterated integral in cylindrical coordinates whose value is equal to the value of this triple integral.
   (b) Set up an iterated integral in spherical coordinates whose value is equal to the value of this triple integral.

   Do not evaluate these iterated integrals.



$$\iiint_{\mathcal{W}} (2x+3y+94z)\,dV = \int_0^{2\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} (2r\cos\theta + 3r\sin\theta + 94z) r\,dz\,dr\,d\theta$$

$$\iiint_{\mathcal{W}} (2x+3y+94z)\,dV = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{8}} (2\rho\sin\phi\cos\theta + 3\rho\sin\phi\sin\theta + 94\rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

5. [14 points] The map $\Phi(u,v) = (3u+4v, u-2v)$ takes the square $\mathcal{D}_0 = [0,1]\times[0,1]$ in the uv-plane to the parallelogram $\mathcal{D}$ with vertices $(0,0)$, $(3,1)$, $(4,-2)$, and $(7,-1)$ in the xy-plane. Use the Change of Variables Formula to evaluate the double integral $\displaystyle\iint_{\mathcal{D}} (x+2y)\,dA$.

Note that $\text{Jac}(\Phi) = \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -10$.

$$\iint_{\mathcal{D}} (x+2y)\,dA = \int_0^1\int_0^1 (3u+4v+2(u-2v))|\text{Jac}(\Phi)|\,du\,dv = 10\int_0^1\int_0^1 5u\,du\,dv = 25$$

6. [10 points] Consider the vector field $\mathbf{F} = \langle e^x, y^2, xz\rangle$.
   (a) Find the vector assigned to the point $P = (0,-2,1)$ by the vector field $\mathbf{F}$.
   (b) Explain why the vector field $\mathbf{F}$ is not a gradient vector field.

   $\mathbf{F}(0,-2,1) = \langle 1,4,0\rangle$ Since $\dfrac{\partial F_1}{\partial z} = 0 \neq z = \dfrac{\partial F_3}{\partial x}$, $\mathbf{F}$ is not a gradient vector field.

7. [16 points] Let $\mathcal{C}$ be the curve parameterized by $\mathbf{c}(t) = (3\cos(t), 3\sin(t), 4t)$, $0 \le t \le \pi$.
   (a) For the function $f(x,y,z) = xy+z$, evaluate the line integral $\displaystyle\int_{\mathcal{C}} f(x,y,z)\,ds$.
   (b) For the vector field $\mathbf{F} = \langle x,y,z\rangle$, evaluate the line integral $\displaystyle\int_{\mathcal{C}} \mathbf{F}\bullet d\mathbf{s}$.

   Note that $\mathbf{c}'(t) = (-3\sin(t), 3\cos(t), 4)$, and that $||\mathbf{c}'(t)|| = 5$.

$$\int_{\mathcal{C}} f(x,y,z)\,ds = \int_0^\pi f(\mathbf{c}(t))||\mathbf{c}'(t)||\,dt = \int_0^\pi (9\sin(t)\cos(t)+4t)\,5\,dt = 10\pi^2$$

$$\int_{\mathcal{C}} \mathbf{F}\bullet d\mathbf{s} = \int_0^\pi \mathbf{F}(\mathbf{c}(t))\bullet\mathbf{c}'(t)\,dt = \int_0^\pi \langle 3\cos(t), 3\sin(t), 4t\rangle \bullet \langle -3\sin(t), 3\cos(t), 4\rangle\,dt$$

$$= \int_0^\pi 16t\,dt = 8\pi^2$$