MATH 4039-1 **Final Review** **Spring 2014**

The final exam will take place on Friday, May 9, 5:30–7:30 pm, in our usual room. In addition to the material covered on the midterm exam, the final exam will also cover material we've discussed from Chapters 3, 4, and 6 in the text (and related material discussed in class). A (likely non-comprehensive) list of concepts etc. regarding this latter material you should know and be ready to work with is below. Refer to the midterm review for an analogous list pertaining to material discussed earlier in the course. If you have questions regarding any of the material, make use of my office hours, and/or ask by email. I anticipate being in my office on Tuesday and Thursday of finals week.

I anticipate that (some of the) problems on the exam will be of a similar nature to those you've encountered in the homework (both collected and uncollected), examples and proofs we've discussed in class, etc.

Some concepts, results, definitions... you should know, be able to prove...

Know the definition of a (2-dimensional) *cell complex*/a *cell decomposition* of a surface. (page 67)

Know what the *Euler characteristic* of a surface is and how to compute it from a cell decomposition(page 67), the fact that it is a topological invariant (Theorem 3.1.7), the Euler characteristic of a connected sum (Theorem 3.1.4).

Be able to determine if two surfaces are homeomorphic using the Euler characteristic and orientability (Cor. 3.1.8).

Know the definition of a *triangulation* of a surface (§3.2), and be comfortable working with triangulations and various properties (e.g., $3f = 2e$).

Know what the *genus* of a surface is (in terms of *handles* or *cross-caps*), its relation to the Euler characteristic, to the maximal number of disjoint nonseparating curves, etc. (§3.3)

Know the definition of a *regular complex* on a surface (§3.4), and be comfortable working with them (and dual complexes) and various properties (e.g., $2e = af$, $2e = bv$, page 82).

Know the definition of a *b-valent complex* on a surface (§3.5), and be comfortable working with them and various properties. What properties does the dual of a b-valent complex have? Be comfortable working with cell complexes of this form too.

Know the definition of a *map* on a surface, what it means to *color* a map, what the *coloring number* of a surface is etc. (§4.1) Be comfortable working with properties of maps (e.g., a map on $\mathbb{S}^2$ has a face with less than six sides).

If there is a positive integer N so that $\dfrac{2e}{f} < N$ for all maps on the surface M, then any map on M can be N-colored (Prop. 4.1.2). Know, and be able to work with the relationship between $2e/f$ and $\chi(M)$. (pages 94–95)

What is the *Haewood number* of a surface M? What is its significance? (pages 95–98)

Be comfortable working with graphs embedded in surfaces (§4.4), Be able to show that a given graph can or cannot be embedded in a given surface (e.g., pages 107–110).

Know the definition of the *genus of a graph*, what it means for a graph to be *minimally embedded* in a surface, and the significance of the latter (Theorem 4.4.14 and surrounding discussion).

What is the relationship between the genus of a graph and the Euler characteristic of a surface in which the graph minimally embeds? What is (a lower bound for) the genus of a complete graph? (§4.5)

Know what a *n-coloring* of a graph is. What is the relationship between map coloring and graph coloring? For a surface other than the Klein bottle, what is the largest complete graph that can be embedded in the surface? How does this relate to map-coloring? (pages 115–117)

Be comfortable working with *paths*, *loops*, and related notions such as *inverse paths*, *products of paths*... (§6.1)

Know the definition of *path homotopy*, and be able to work with this notion. (page 154)

The path homotopy relation is an equivalence relation. (Prop. 6.1.5)

Know the definition of the *fundamental group* of a (path connected) topological space. What are the elements? What is the group operation? What is the identity? What are inverses? (pages 156–158)

Know what a *simply connected* space is. Show that a (star) convex subset of $\mathbb{R}^n$ is simply connected. (page 159)

A continuous map between spaces induces a homomorphism between fundamental groups. (Prop. 6.1.17)

The fundamental group of the circle is infinite cyclic. Be comfortable working with loops in $\mathbb{S}^1$ and lifts of these to paths in $\mathbb{R}$. What is the isomorphism from $\pi_1(\mathbb{S}^1)$ to $\mathbb{Z}$? (§6.2)

If $A \subset X$ is a subspace and $f, g \colon X \to Y$ are continuous, know what it means for f and g to be *homotopic relative to A*. Know what it means for A to be a *deformation retract* of X. (§6.3)

If $A \subset X$ is a deformation retract, then the fundamental groups of A and X are isomorphic. (Theorem 6.3.6)
Be prepared to use this result.

Be prepared to use Theorem 6.4.2, which gives conditions under which a topological space is simply connected.
For instance, $\mathbb{S}^n$ is simply connected for $n \geq 2$; $\mathbb{R}^n \setminus \mathbf{0}$ is simply connected for $n \geq 3$ (why?).