1. An absent-minded professor does not remember which of his 8 keys opens his office door. If he tries them at random, with replacement, what is the probability that he will need 3 tries to open his door?

   If X is the number of tries to open the door, then X is a geometric random variable with $p = 1/8$. $P(X = 3) = (7/8)^2(1/8) = 0.0957$.

2. A fair die is rolled 10 times independently. Find the probability that a 3 is rolled at least twice.

   If X is the number of times 3 is rolled, then X is a binomial random variable with $n = 10$ and $p = 1/6$. $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) = 1 - \binom{10}{0}(5/6)^{10} - \binom{10}{1}(1/6)(5/6)^9 = 0.5155$

3. In a certain textbook, one misprint is expected every 5 pages. If there is at least one misprint on a given page of this book, what is the probability that there are exactly 3 misprints on that page?

   If X is the number of misprints on the page, then X is (approximately) a Poisson random variable with $\lambda = 1/5$. $P(X = 3 \mid X \geq 1) = P(X = 3)/P(X \geq 1) = P(X = 3)/(1 - P(X = 0)) = (e^{-\lambda}\lambda^3/3!)/(1 - e^{-\lambda}) = 0.0060$

4. A person arrives at a bus stop at 10 o'clock, knowing that the bus will arrive at a random time between 10 and 10:30. What is the probability that the person will have to wait at least 10 minutes?

   If X is the number of minutes after 10 o'clock when the bus arrives, then X is a uniform random variable with probability density function $f(x) = \begin{cases} 1/30 & \text{if } 0 \leq x \leq 30, \\ 0 & \text{otherwise.} \end{cases}$ $P(X \geq 10) = \int_{10}^{30} f(x)\,dx = 2/3$

5. A certain discrete random variable X has possible values 0, 2, 4, 6, and 8. The probability mass function of X satisfies $p(0) = .05$, $p(2) = .1$, $p(4) = .25$, and $p(8) = .37$.
   (a) Find $P(X \leq 4)$. $P(X \leq 4) = .40$
   (b) Find $P(X = 6)$. $P(X = 6) = .23$
   (c) Is the expected value of X greater than 4, less than 4, or equal to 4? $E[X] > 4$

6. Suppose that X is a random variable with $E[X] = 5$ and $\mathrm{Var}(X) = 3$.
   (a) Find $E[X^2]$. $E[X^2] = 28$
   (b) Find $E[2X - 9]$. $E[2X - 9] = 1$
   (c) Find $\mathrm{Var}(2X - 9)$. $\mathrm{Var}(2X - 9) = 12$

7. A multiple choice test has 100 questions, each with possible answers a, b, c, d, e. Estimate the probability that with random guessing, the number of correct answers is at least 30.

   If X is the number of correct answers, then X is binomial with $n = 100$ and $p = 1/5$. Note that $np = 20$ and $np(1 - p) = 16$. Normal approximation yields $P(X \geq 30) = 1 - P(X < 30) = 1 - P(X \leq 29.5) = 1 - P((X - 20)/4 \leq 2.375) \approx 1 - \Phi(2.375) = 0.0088$

8. The probability density function of a random variable X is given by

$$f(x) = \begin{cases} x^3/20 & \text{if } 1 \leq x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the cumulative distribution function $F(x)$ of the random variable X.

$$F(t) = \int_{-\infty}^{t} f(x)\,dx = \begin{cases} 0 & t \leq 1 \\ (t^4 - 1)/80 & 1 < t < 3 \\ 1 & t \geq 3 \end{cases}$$

(b) Find the probability density function of the random variable $Y = X^2$.

$$P(Y \leq t) = P(X^2 \leq t) = P(X \leq \sqrt{t}) = \begin{cases} 0 & t \leq 1 \\ (t^2 - 1)/80 & 1 < t < 9 \\ 1 & t \geq 9 \end{cases}$$

So $f_Y(t) = \begin{cases} t/40 & 1 < t < 9 \\ 0 & \text{otherwise} \end{cases}$

9. Consider the random variable X and probability density function $f(x)$ from problem 8.

   (a) Find $P(X \leq 2)$. $P(X \leq 2) = \int_1^2 x^3/20 \, dx = 0.1875$

   (b) Find the expectation $E[X]$. $E[X] = \int_1^3 x \cdot x^3/20 \, dx = 2.42$

   (c) Find the expectation of the random variable $Y = X^2$. $E[X^2] = \int_1^3 x^2 \cdot x^3/20 \, dx = 6.0667$

10. Let Z be the standard normal random variable.

   (a) Find $P(0 \leq Z \leq 3.14)$. $P(0 \leq Z \leq 3.14) = \Phi(3.14) - \Phi(0) = 0.4992$

   (b) Find $P(Z > 2.71)$. $P(Z > 2.71) = 1 - P(Z \leq 2.71) = 1 - \Phi(2.71) = 0.0034$

   (c) Find the value of a for which $P(-a \leq Z \leq a) = 0.754$. $a = 1.16$