

1023-14-1068

Benjamin J Howard* (bhoward@ima.umn.edu), I.M.A., University of Minnesota, 400 Lind Hall, 207 Church St SE, Minneapolis, MN 55455, and **John Millson** (jjm@math.umd.edu), **Andrew Snowden** (asnowden@math.princeton.edu) and **Ravi Vakil** (vakil@math.stanford.edu). *The space of n ordered points on the line is cut out by simple quadrics if n is not six.*

We study the projective invariants of n labelled points on the projective line – that is, polynomials in the homogeneous coordinates X_i, Y_i ($1 \leq i \leq n$) which are invariant under the diagonal action of $SL(2)$. We shall assume the i -th point has weight w_i . Let R_w denote the graded ring of projective invariants, where $w = (w_1, \dots, w_n)$.

By the classical theorem of Kempe (1894) we know that R_w is generated in degree one for any number of points n and weighting w , provided that the total weight $w_1 + \dots + w_n$ is even. (If the total weight is odd, then R_w is zero in odd degree components, so we exclude this case for simplicity.) The generators correspond to directed multigraphs with vertex set $\{1, \dots, n\}$ such that $\deg(i) = w_i$ for each i .

We show that $\text{Proj}(R_w)$ is cut out scheme-theoretically by linear and quadric relations in the above graphs, unless $n = 6$ and each $w_i = 1$. We show by other means that the ideal of relations is generated in degree ≤ 4 , for any n and w . (Received September 25, 2006)