Thrusday, 9:00–10:00

Room 148

Permanents and Edge-Colouring

Alexander Schrijver, CWI and University of Amsterdam

The permanent of an $n \times n$ matrix $A = (a_{i,j})$ is defined by

$$\mathrm{per}(A) := \sum_{\pi} \prod_{i=1}^{n} a_{i,\pi(i)},$$

where π ranges over all permutations of 1, 2, ..., n.

Van der Waerden (1926) asked if the permanent of any doubly stochastic $n \times n$ matrix is at least $n!/n^n$, which was proved in 1981 by Falikman.

Related is the question of Erdős and Rényi (1968) for the maximum value α_k such that $\mathrm{per}(A) \geq \alpha_k^n$ for each nonnegative integer $n \times n$ matrix A with each row and column sum equal to k. So α_k^n is a lower bound on the number of 1-factors in a k-regular bipartite graph on $2n$ vertices.

Voorhoeve found in 1978 that $\alpha_3 = \frac{4}{3}$. Recently we found the exact value of α_k for general k. It implies the currently best lower bound 0.44007584 for Kasteleyn's dimer problem in 3 dimensions.

The methods also imply an $O(km)$ time algorithm to find a perfect matching in a k-regular bipartite graph. This gives an $(m\Delta)$ time algorithm for colouring the edges of a bipartite graph, sharpened by Cole, Ost, and Schirra to $O(m \log \Delta)$ (m = number of edges, Δ = maximum degree).

In the lecture we give an introduction to the results and methods.

1:30–2:30

Room 148

Graph Embedding and Eigenvalues

Alexander Schrijver, CWI and University of Amsterdam

In 1990, Colin de Verdiére characterized planar graphs by means of a graph parameter $\mu(G)$ based on the largest multiplicity of the second eigenvalue of matrices associated with a graph G: $\mu(G) \leq 3$ if and only if G is planar. The parameter is motivated by estimating the multiplicity of the second eigenvalue of of Schrödinger operators on dRiemann surfaces.

With L. Lovász we proved in 1998 that $\mu(G) \leq 4$ if and only if G is linklessly embeddable. The proof is based on a Borsuk theorem for antipodal links, that might be of independent interest.

Recent results of Lovász suggest a close connection between the matrices associated with a graph, and its representation as the skeleton of a convex polytope.

In the lecture, we give an introduction to the above, and we explain the methods.