

**Algebra/Geometry Session Problems**  
**Questions 1-20 multiple choice**

- Answer only one choice: (a), (b), (c), (d), or (e) for each of the following questions.
- Only use a number 2 pencil.
- Make heavy black marks that fill the circle.
- Erase clearly any answer you wish to change.
- Do not make stray marks on the answer sheet.

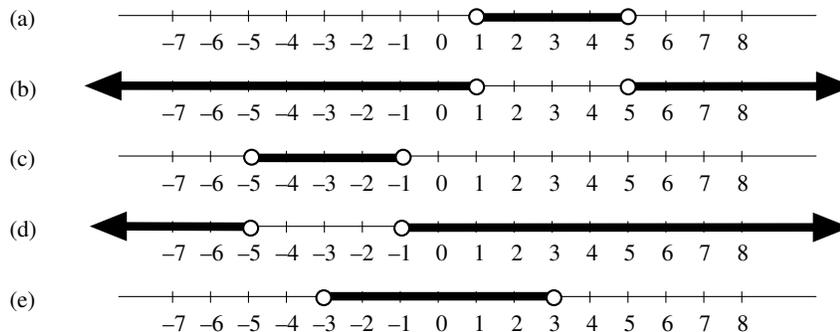
1. Which of the following equations is the equation of a line parallel to  $y = \frac{2}{3}x + 16$ ?

- (a)  $y + \frac{2}{3}x = 7$
- (b)  $2y + 3x = 7$
- (c)  $6y + 4x = 7$
- (d)  $6y - 4x = 7$
- (e)  $2y - 3x = 7$

2. Solve for  $x$  in  $3x + 7 = 1 - 2x$ .

- (a)  $-\frac{6}{5}$
- (b)  $\frac{1}{5}$
- (c)  $\frac{6}{5}$
- (d)  $-\frac{7}{6}$
- (e)  $\frac{7}{6}$

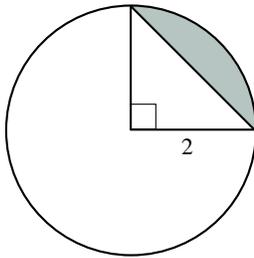
3. Which of the following is the graph of the set of  $x$  satisfying the inequality  $|x + 3| < 2$ ?



4. The roots of  $(x^2 - 3x + 2)(x)(x - 4) = 0$  are:

- (a) 4
- (b) 0 and 4
- (c) 1 and 2
- (d) 0,1,2 and 4
- (e) 1,2 and 4

5. Find the area of the shaded region in the diagram below.



- (a)  $\frac{\pi-1}{2}$
- (b)  $\pi - 2$
- (c)  $\frac{\pi-2}{2}$
- (d)  $2\pi - 4$
- (e)  $\frac{\pi}{2} + 4$

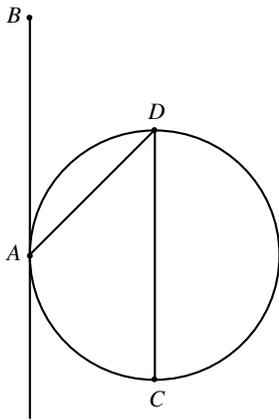
6. An equation of the line through  $(2, 4)$  with slope 3 is

- (a)  $y - 4 = 3(x - 2)$
- (b)  $y - 2 = 3(x - 4)$
- (c)  $y = 3x - 4$
- (d)  $y = 3x + 4$
- (e) None of the above.

7. If  $x = 3$  and  $y = -2$ , then  $\frac{xy}{x-y}$  is

- (a) 6
- (b) -1
- (c) 5
- (d)  $-\frac{6}{5}$
- (e)  $\frac{6}{5}$

8. What is the midpoint of the line segment that runs between  $(-3, 1)$  and  $(1, 5)$ ?
- (a)  $(2, -2)$   
 (b)  $(-2, 2)$   
 (c)  $(-2, 6)$   
 (d)  $(2, -6)$   
 (e)  $(-1, 3)$
9. In the diagram below  $\overline{AB}$  is parallel to  $\overline{CD}$ ,  $\overline{CD}$  is a diameter and  $\overline{AB}$  is a tangent line. Find the measure of  $\angle DAB$  in degrees.



- (a) 20  
 (b) 30  
 (c) 45  
 (d) 60  
 (e) None of the above.
10. Three times the measure of an angle is the measure of its supplement. What is the degree measure of the angle?
- (a) 30  
 (b) 45  
 (c)  $\frac{45}{2}$   
 (d) 60  
 (e) None of the above.

11. Which of the following statements are true?

(I):  $-1 < x < 2$  implies  $0 < x^2 < 4$

(II):  $-1 < x < 2$  implies  $-1 < x^2 < 4$

(III):  $-1 < x < 2$  implies  $1 < x^2 < 4$

(a) Only I.

(b) Only II.

(c) Only III.

(d) Both I and II.

(e) All of I, II, and III.

12. A checkerboard has squares which are 2 inches on each side. A coin with diameter 1 inch is tossed into the air and lands with its center on the board. What is the probability that the coin doesn't touch the edge of any square?

(a)  $\frac{1}{8}$

(b)  $\frac{1}{4}$

(c)  $\frac{\sqrt{2}}{4}$

(d)  $\frac{1}{2}$

(e)  $\frac{\sqrt{2}}{2}$

13. Completely factor the polynomial  $x^3 - 2x^2 - 2x$ .

(a)  $x(x^2 - 2x - 2)$

(b)  $x(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})$

(c)  $x(x + 1 + \sqrt{3})(x + 1 - \sqrt{3})$

(d)  $x(x - 2)(x + 1)$

(e) None of the above is correct.

14. If the length of a rectangle is increased by 30% and the width is decreased by 20%, then the area is

(a) increased by 4%

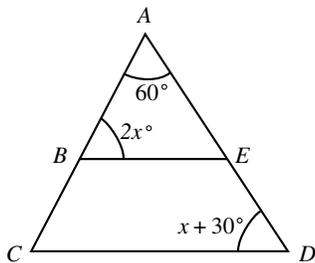
(b) decreased by 6%

(c) increased by 6%

(d) increased by 10%

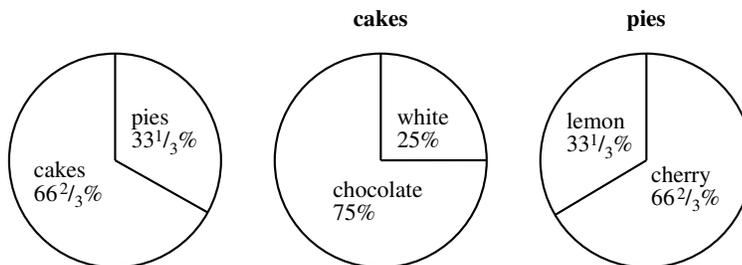
(e) increases by 56%

15. Suppose the equation  $x^2 + bx + 4 = 0$  has exactly one solution. The best statement about  $b$  is
- $b$  must be equal to 4
  - $b$  must be equal to -4
  - $b$  must be equal to 2
  - $b$  must be equal to -2
  - none of the above
16. Of the following statements, the one that is incorrect is:
- Doubling the base of a rectangle doubles the area.
  - Doubling the altitude of a triangle doubles the area.
  - Doubling the radius of a circle doubles the area.
  - Doubling the denominator of a positive fraction and dividing its numerator by 2 changes the fraction.
  - Doubling a given number may make it less than it originally was.
17. In the diagram below  $\overline{BE}$  and  $\overline{CD}$  are parallel. If  $\angle CAD$  measures  $60^\circ$ ,  $\angle ABE$  measures  $2x$  degrees, and  $\angle CDE$  measures  $x + 30$  degrees, then find  $x$ .



- 15
  - 30
  - 45
  - 60
  - None of the above
18. During the final game of a basketball tournament, only 7 players from the tournament winning team played. The scoring average of all 7 players was 13. The scoring average of everyone but the point guard was 12. How many points did the point guard score?

- (a) 14  
 (b) 16  
 (c) 19  
 (d) 21  
 (e) 24
19. A drawer contains 10 red socks, 6 white socks, 8 blue socks, and 7 purple socks. It is too dark to see so you just start grabbing socks hoping to get a pair of the same color. How many socks do you need to grab to guarantee that you get such a matching pair?
- (a) 2  
 (b) 5  
 (c) 7  
 (d) 9  
 (e) 11
20. A bakery sells cakes and pies. It sells lemon and cherry pie. It sells chocolate and white cake. These sales are represented on the pie charts below.



Consider the following statements:

- (I): The bakery sells more white cake than cherry pie.  
 (II): The bakery sells more chocolate cake than pie.

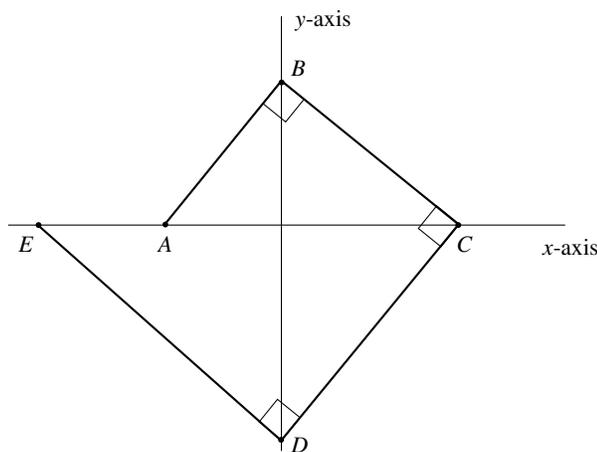
Which of the following statements is correct about the relationship of sales of baked goods.

- (a) Only I is true.  
 (b) Only II is true.  
 (c) Both I and II are true.  
 (d) Neither I nor II are true.  
 (e) More information is required to determine the relationship.

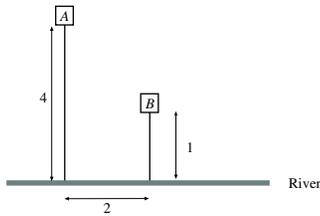
### Exact Answer Questions

These next ten questions are exact numerical answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for  $\pi$  or other irrational numbers. Answers must be exact.

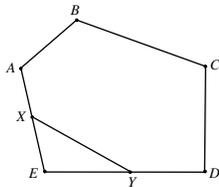
1. An airport has a moving sidewalk. Standing on the moving sidewalk, Joe takes 3 minutes to reach the end. Walking next to the moving sidewalk, Joe takes 4 minutes to reach the end. How long does it take Joe to reach the end if he walks on the moving sidewalk?
2. John earned some money. When he was paid, his employer had already deducted 20% federal and 2% state income taxes. He was paid \$792. How much, to the nearest dollar, did he earn?
3. A cube is 100 inches on each side. The cube is painted blue and then cut into 1,000,000 small cubes with 1 inch sides. How many of the small cubes have no blue paint on them?
4. A six place number is formed by repeating a three place number: For example, 256,256, or 678,678, etc. Call these numbers copycats. Find greatest common divisor of all the copycats.
5. How many diagonals can be drawn in a polygon of 100 sides?
6. In the diagram below,  $A$  is the point  $(-1, 0)$ . Suppose that  $\angle ABC = \angle BCD = \angle CDE$  all measure  $90^\circ$ , and  $\overline{EA} = 2\overline{AC}$ . Find the slope of the line containing  $\overline{AB}$ .



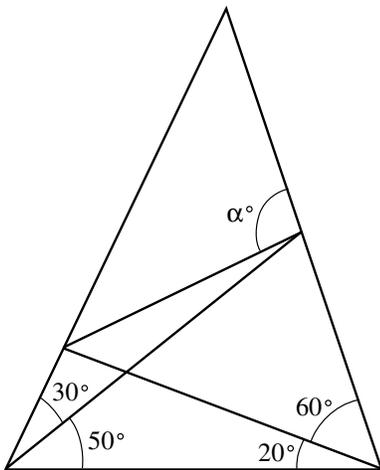
7. A river flows along a straight line. House A is 4 miles from the river and house B is 1 mile from the river. The house A is 2 miles down river from house B. You are to take a bucket from house A, get water from the river, and bring it to house B. What is the shortest distance that you are required to travel?



8. In the diagram, the area of the quadrilateral ABCD is 75% of the area of the pentagon ABCDE, X bisects  $\overline{AE}$ , and Y bisects  $\overline{ED}$ . If the area of the triangle XYE is 1, then what is the area of the pentagon ABCDE.



9. The graph of  $y = f(x)$  is a line of slope 3. What is the slope of the graph of  $y = f(2x + 1)$ ?
10. Consider the triangle below. Find the angle  $\alpha$  in degrees.



### **Tie Breaker Question**

This last page is the tiebreaker question. You should give an argument. It is an essay question, *i.e.*, it is graded for its clarity as well as its correctness. It is graded only to separate first, second and third place ties. It is the only question graded for partial credit.

Question: Find all pairs of integers whose sum equals their product. Justify that your list contains all solutions.