2011 LSU Math Contest
Open Session

Questions 1 - 14 are worth 1 point each and questions 15 - 25 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

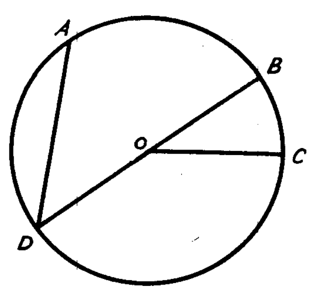
Questions 1 - 14 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1
Two chords intersect inside a circle. One chord is divided into segments 6 and 8 units long by the second chord. If one segment of the second chord is 4, the length of the other segment is

A 10 B 12 C 16 D 18 E None of these

2
If the figure, O is the center of a circle of radius 3, the points D, O and B are collinear. If $\angle ADB$ = 44° and $\angle BOC$ =32°, the length of the arc $\overset{\frown}{\text{ABC}}$ is



A 3π B $3\pi\cos(76°)$ C $3\pi/2$ D $\frac{\pi}{2}\cos(76°)$ E 2π

3
A circle is inscribed in a right triangle having sides 3, 4, and 5. The area of the inscribed circle is

A $25\pi/4$ B 4π C π D 9π E None of these

4
The fourth term of the arithmetic progression $(3-\sqrt{2}), 2, \ldots$ is

A $3+2\sqrt{2}$ B $2\sqrt{2}$ C $3-2\sqrt{2}$ D $3\sqrt{2}$ E $1+\sqrt{2}$

5
A regular octagon is inscribed in a circle of radius 4. The area of the octagon is

A 16π B $128\sqrt{2}$ C $32\sqrt{2}$ D $8\sqrt{2}$ E None of these

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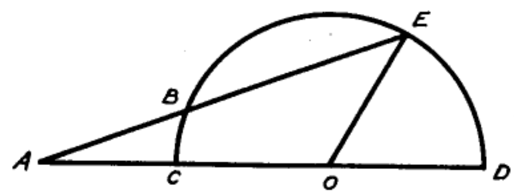
6
The difference between two numbers is 2. Their product is 84 greater than the square of the smaller number. The sum of the numbers is

A 21 B 42 C 84 D 86 E 164

7
In the figure,



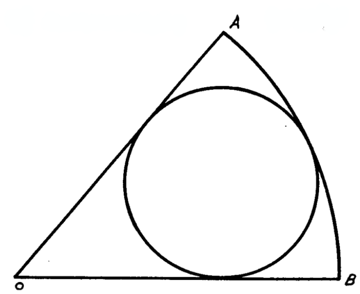
CD is the diameter of a semicircle $\overset{\frown}{\text{CBED}}$ with center O, and $AB = OD$. If $\angle EOD = 60°$, then $\angle BAC$ is

A 15° B 20° C 30° D 45° E None of these

8
The roots of $(x+m)^2 + 2x = 0$ are equal if m is

A 1 B -1 C $-1/2$ D -2 E None of these

9
A circular sector OAB has a radius R and a central angle θ.



The area of the inscribed circle is

A $\pi R^2\cos^2(\theta/2)$ B $\pi R^2\sin^2(\theta/2)$ C $\pi R^2(1-\cos(\theta/2))^2$

D $\dfrac{\pi R^2(1-\cos(\theta/2))}{(1+\sin(\theta/2))^2}$ E $\dfrac{\pi R^2\sin^2(\theta/2)}{(1+\sin(\theta/2))^2}$

10
A crew rows four miles downstream and back the same distance in one hour. If the stream flows at three miles per hour, the crew's rate of rowing in still water is

A 4 mph B -1 mph C 8 mph D 12 mph E 9 mph

11
If $\arctan x + \arctan k = 45°$, then x is

A $1+k$ B $1-k$ C $1-k^2$

D $\dfrac{1-k}{1+k}$ E none of these

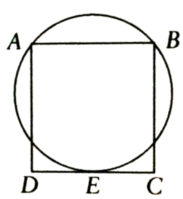
12
Three roots of the equation $2x^4 - 3x^3 + 6x^2 - 12x - 8 = 0$ are $2i$, $-2i$, and 2. The fourth root is

A 1 B -1 C $1/2$ D $-1/2$ E None of these

13
The number of real solutions to the system of equations $16x^2 + 25y^2 = 360$ and $xy = 1$ is

A 0 B 1 C 2 D 3 E 4

14 [*This is also a tie breaker question* - see page 3.]
A square $ABCD$ is drawn so that its side CD is tangent to the circle ABE.



The ratio of the area of the circle ABE to the area of the square $ABCD$ is

A $25\pi : 64$ B $5\pi : 8$ C $\sqrt{5\pi} : 2\sqrt{2}$ D 5 : 8

E Cannot be determined

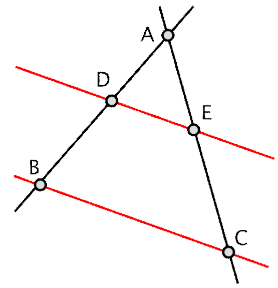
Questions 15 - 25 Exact Answers

These next eleven questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ *or* 6^{40}.

15 To number the pages of a bulky volume the printer used 1890 digits. How many pages has the volume?

16 Find $\tan(1050°)$.

17 In the figure



the area of triangle ADE equals the area of the trapezoid $EDBC$. If $BC = 12$, then $DE =$

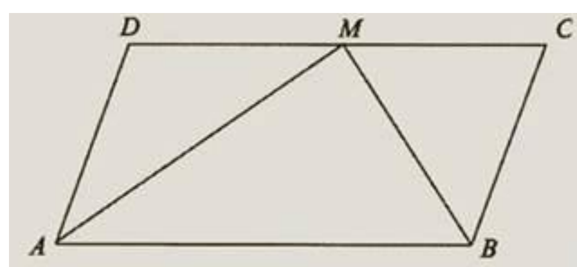
18 A rhombus has a side of length 12 and an angle of 60°. Find its area.

19 If θ is an angle between 0° and 180° and $\tan\theta = -2$, find $\cos\theta$.

20 Find x if a and b are positive numbers with $ab \neq 1$ and $a^x = b^{1-x}$.

21 Assuming that the radius of the earth is 3000 miles, how far does a person travel on the surface if, at latitude 30° she moves 10° of longitude?

22 On the figure, $ABCD$ is a parallelogram, $\angle BAD = 60°$, AM and BM are angle bisectors of angles BAD and ABC respectively. If the perimeter of $ABCD$ is 6 cm, find the sides AB, AM, and BM of triangle ABM.



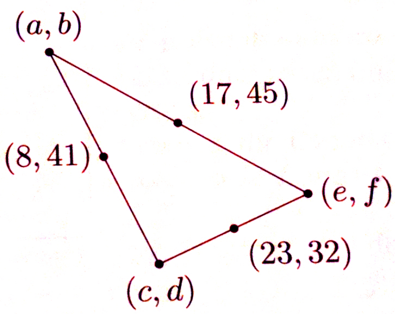
23 A function f is such that for each positive integer n,

$$f(n+1) = \frac{f(n)}{1+af(n)},$$

where a is a real number, $f(1) = 1$, and $f(11) = \dfrac{1}{2011}$. Find a.

24 Find all positive integers n such that $2011 + n^2$ is the square of a positive integer.

25 The points (23, 32), (8, 41), and (17, 45) are the midpoints of the sides of a triangle. Find the largest possible value of $x + y$ where (x, y) is a vertex of the triangle.



Tie Breaker requiring Full Solution

Please give a **detailed explanation** on the answer sheet **of** your **solution to Question 14**.

This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!