2012 LSU Math Contest
Team Problems

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and fifteen minutes to complete the entire team session.

These 10 questions (except 5 and 10) require exact numerical or algebraic answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact.
The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

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1 Given that $\tan\alpha + \cot\alpha = 4$,

find $\sqrt{\tan^2\alpha + \cot^2\alpha}$.

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2 Find the probability that a sum of two randomly picked (different) numbers from the set $\{1, 2, \ldots, 9\}$ is even.

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3 What is the remainder when you divide $x^{2012} + x + 1$ by $x^2 + 1$?

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4 What is the angle between a plane intersecting two parallel faces of cube along their diagonals with a plane intersecting two other parallel faces of a cube along their diagonals?

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5 Positive numbers a and b satisfy the condition

$$\sqrt{a} - \sqrt{b} > a - b > 0.$$

Which of the following three statements about a and b are then true:
A $1 > \sqrt{a} + \sqrt{b}$; B $\sqrt{b} > b$; C $\sqrt{a} > b$.

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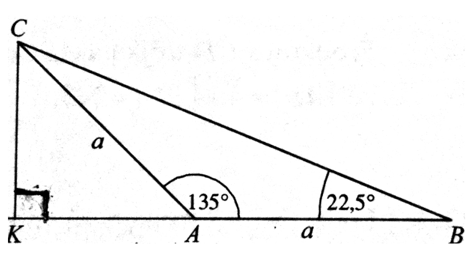
6 Use the triangles on the picture to compute $\tan 22.5°$.



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7 Each face of a cube is to be colored red, yellow or green, such that there are two faces of each color. What is the maximum number of distinctly colored cubes, independent of how they are held?

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8 For twenty consecutive days water has been added to a pool containing originally 1000 m^3 of water. The first day 25 m^3 was added. Then every next day 2 m^3 more water was being added than the previous day. At the same time 50 m^3 has been drained from the pool every day.
After how many days will the amount of water in the pool be the lowest?

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9 Simplify the expression

$$\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}} \times \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}.$$

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10. Divide the pictured polygon into two connected congruent subpolygons both consisting of little triangles. Shade one of the subpolygons.