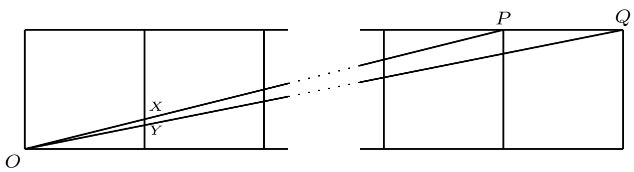
- No calculators are allowed.
- Pictures are only sketches and are not necessarily drawn to scale or proportion.
- You have one hour and fifteen minutes to complete the entire team session.

These 9 problems (except problem **5***) require exact numerical or algebraic answers. Exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers.*

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

1. A number of unit squares are placed in a line as shown in the diagram below.



Let O be the bottom left corner of the first square and let P and Q be the top right corners of the 2012th and 2013th squares respectively. The lines OP and OQ intersect the right side of the first square at X and Y, respectively. Determine the area of triangle OXY .

2. Find a way to cut a 4×9 rectangle into two pieces and when repositioned form a 6×6 square.

3. How many positive integer divisors does $12!$ have? (include $12!$ and 1).

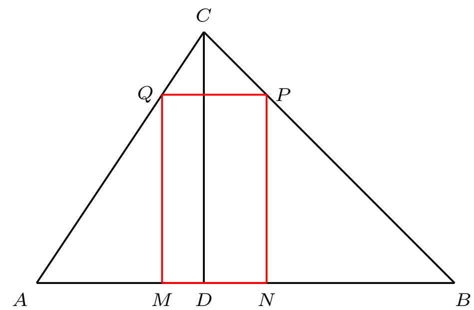
4. If m and n are positive integers such that

$$m^3 - n^3 = 485,$$

find $m^3 + n^3$.

5. Two circles, C_1 and C_2 with radii r_1 and r_2, respectively, $r_1 < r_2$, are tangent to each other and C_1 is not contained in C_2. A line is tangent to both circles at the points D_1 and D_2 with $|D_1D_2| = d$.
Which of the following three statements about r_1, r_2 and d are then true:
A $d < r_1 + r_2$; B $2r_1 < d < 2r_2$; C $d = 2\sqrt{r_1 r_2}$.

6. In the picture below, $|AB| = 12$, $|CD| = 6$, and the area of the rectangle $MNPQ$ is 10.



Find the lengths of the sides of the rectangle $MNPQ$.

7. The Math Club needs to choose a committee consisting of two girls and two boys. If the committee can be chosen in 3630 ways, how many students are there in the Math Club?

8. Compute the following sum

$$S = \frac{1}{7^0} + \frac{1}{7^1} + \frac{2}{7^2} + \frac{3}{7^3} + \frac{5}{7^4} + \frac{8}{7^5} + \cdots,$$

where the numerators are the Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, \ldots$ and the denominators are the successive powers of 7. Your answer should be a rational number.

9. If a man walks to work and rides back home it takes him an hour and a half. When he rides both ways it takes 30 minutes.
How long would it take him to make the round trip by walking?