

Difference Equations: An Overview

Many students are somewhat fascinated by the Fibonacci Sequence:

$$0, 1, 1, 2, 3, 5, 8, \dots,$$

where any entry is the sum of the previous two. This pattern is found in many natural phenomenon and for this reason the Fibonacci sequence is called ‘Nature’s numbering system’. We can write an equation to describe this sequence as

$$a_{n+2} = a_n + a_{n+1},$$

with the first two, $a_0 = 0$ and $a_1 = 1$, specified. In this talk we will see that a_n may be solved in closed form as

$$a_n = \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1 - \sqrt{5}}{2} \right)^n.$$

The equation $a_{n+2} = a_{n+1} + a_n$ belongs to a broad class of equations called **difference equations**. The purpose of this talk is to discuss how to solve such equations in closed form. Some of the tools needed are:

- Finding roots of polynomials or factoring
- working with radicals
- working with complex numbers

Difference equations provides a nice little application for those students who ask ‘why do I need to know how to do these things’.