- No calculators are allowed.
- Pictures are only sketches and are not necessarily drawn to scale or proportion.
- You have one hour and fifteen minutes to complete the entire team session.

These 10 problems require exact numerical or algebraic answers. Exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

1. The number of seconds in 6 weeks is $n!$. Find n.
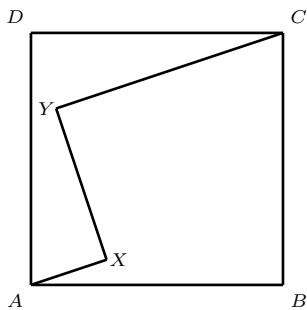
2. Suppose f is a real valued function such that
$$f(m + 1) = m(-1)^m - 2f(m)$$
and $f(1) = f(2015)$. Find
$$f(1) + f(2) + \cdots + f(2014).$$

3. Find the largest number p so that 910 is the sum of p consecutive positive integers.

4. In the sketch below ABCD is a square, AX has length 1, XY has length 2, and YC has length 3. Furthermore, $\angle AXY = 90°$ and $\angle XYC = 90°$. Determine the area of the square.


5. Positive integers can be filled into the following grid so that each row and each column is an arithmetic progression. What is the number that must occupy the square marked by the star ($\star$)?

6. Suppose x and n are positive integers such that $x^2 + 847 = 2^{2n}$. Determine $x + n$.

7. In the diagram below two concentric circles are shown with radii 4 and 8. The point A is the midpoint of OB. The line segment AC has length 5. Determine the length of the line segment CB.
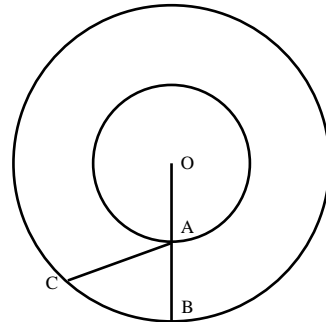
8. If the letters in the word BANANA are rearranged what is the probability that at least one of the six positions will be occupied by the same letter as in the original word?

9. Let S be the set of all 5-digit numbers formed from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ without repetition. Find the number of integers in S that have 3, 5, and 7 in order (not necessarily consecutively) from left to right. For example, 93157 is such number.

10. $\triangle ABC$ is an isosceles triangle with the length of sides AB and AC equal. Suppose D is a point on side AB and E is a point on side AC such that DE and BC are parallel. Further, suppose triangle ADE and trapezoid BCED have equal area and equal perimeter. Find $\cos \angle ABC$. The following diagram may be helpful.