

Questions 1 - 13 are worth 1 point each and questions 14 - 24 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

The people supervising this test are not permitted to explain to you the meaning of any question.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 13 Multiple Choice

Please:

- Use the answer sheet for your answers.
- Answer only one choice A, B, C, D, or E for each question by writing your answer on the answer sheet.
- Completely erase any answer you wish to change.
- Do not make stray marks on the answer sheet.

1. A garden is rectangular in shape with width twice the length. It is surrounded by a 3 feet wide walk way with area 252 square feet. What is the area of the garden in square feet?

- A 108 B 144 C 168 D 288 E 432

2. A polynomial $P(x)$ satisfies $P(x^2 + 1) = x^4 - 2x^2 + 1$. Find $P(x^2 + 2)$.

- A x^4 B $x^4 - 2x^2 + 2$ C $x^4 + 2$ D $x^4 + 2x^2 + 4$ E $x^2 + 2$

3. How many positive even integers are there less than 1000 whose digits multiply to 168?

- A 1 B 2 C 4 D 6 E 8

4. In southern France a common greeting between two individuals is to cheek kiss 3 times. Suppose there are 50 people in a room and they greet each other according to this common custom. How many cheek kisses take place?

- A 150 B 1625 C 1800 D 3675 E 7500

5. Suppose $\cos 2x = \frac{3}{5}$. What is $\cos^4 x - \sin^4 x$?

- A $\frac{3}{5}$ B $\frac{4}{5}$ C $\frac{-3}{5}$ D $\frac{-4}{5}$ E $\frac{4}{7}$

6. A box contains 5 red balls and 9 black balls. You randomly draw two balls without replacement. What is the probability that at least one is red?

- A $\frac{10}{91}$ B $\frac{25}{91}$ C $\frac{5}{13}$ D $\frac{45}{91}$ E $\frac{55}{91}$

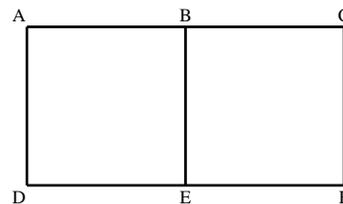
7. The sequence of digits

1234567891011121314...

is obtained by writing the integers 1, 2, 3... one after the other. Notice that the 10th digit is 1 and the 15th digit is 2. What is the 2018th digit?

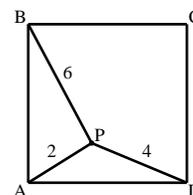
- A 0 B 1 C 3 D 7 E 9

8. Rectangle $ACFD$ is made from two squares $ABED$ and $BCFE$ which share the common side BE . See the illustration below. If the length of \overline{DB} is 6 what is the length of \overline{DC} ?



- A 12 B $3\sqrt{5}$ C $3\sqrt{10}$ D 9 E $6\sqrt{2}$

9. A point P can be identified on the interior of rectangle $ABCD$ such that $|\overline{AP}| = 2$, $|\overline{BP}| = 6$, and $|\overline{DP}| = 4$. Find $|\overline{CP}|$. See the figure below.



- A $6\sqrt{2}$ B $4\sqrt{3}$ C $8\sqrt{2}$ D $6\sqrt{3}$
E Cannot be uniquely determined

10. A point P can be identified on the interior of rectangle $ABCD$ such that $|\overline{AP}| = 2$, $|\overline{BP}| = 6$, and $|\overline{DP}| = 4$. Find the area of the rectangle. See the figure for the previous problem.

- A $6\sqrt{2}$ B $4\sqrt{3}$ C $8\sqrt{2}$ D $6\sqrt{3}$
E Cannot be uniquely determined

11. Which of the following numbers is the largest?

- 3^{30} , 4^{25} , 5^{20} , 6^{15} , 7^{10}
A 3^{30} B 4^{25} C 5^{20} D 6^{15} E 7^{10}

12. What is the highest power of 39 that divides 2018!?

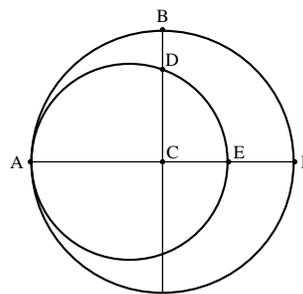
- A 51 B 155 C 156 D 166 E 207

13. In Mr Moore's discrete math class there are 40 students. There are 10 students who major in mathematics and 16 students who major in chemistry. There are 3 who major in mathematics and physics, 4 students who major in chemistry and mathematics, and 5 students who major in physics and chemistry. There is 1 student who majors in mathematics, physics and chemistry, while there are 3 students who do not major in any of these subjects. How many major in physics but not math?

- A 19 B 22 C 23 D 26 E Cannot be determined.

Questions 14 - 24 Exact Answers

These next ten questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40} .



14. What is the angular velocity of the hour hand on a standard 12-hour clock? Express your answer in degrees per minute.

15. You roll a fair die three times and add the numbers that come up. What is the probability that the sum is 5?

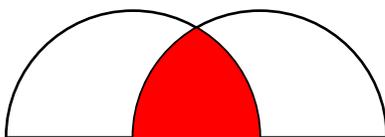
16. A staircase has 10 steps. How many ways are there to ascend the staircase if you either take one step at a time or three steps at a time?

17. Find the largest number n so that

$$1 + 2 + 3 + \dots + n < 1000.$$

18. The power seven lottery awards prize money in powers of 7. For example, prize categories are $1 = 7^0$, $7 = 7^1$, $49 = 7^2$, $343 = 7^3$, etc. In this lottery there are several awards that amount to 777, 777 and there are no more than 6 awards for each prize category. How many awards are there?

19. Two semicircles have radius r with distance r between their centers. Their diameters are on the same line. Find the area of intersection.



20. How many positive integers n are there such that the remainder is 2 when 2018 is divided by n .

21. In the diagram below the smaller circle lies inside the larger circle and is tangent at A. The center of the larger circle is C. Suppose $\overline{BD} = 12$, and $\overline{EF} = 18$. What is the radius of the larger circle?

22. The following series is equal to what fraction?

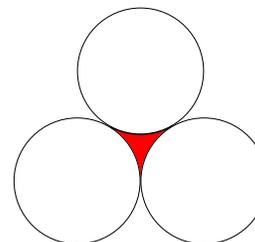
$$\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \dots$$

23. What is the remainder when

$$x^{2018} - x$$

is divided by $x^2 - 1$?

24. Each of the circles in the illustration below have radius 1. They are mutually tangent. What is the area of the shaded region?



Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 16**. Write your explanation on the **reverse side** of your answer sheet.

This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.