Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

**Questions 1 - 21 Multiple Choice**

*Please:*
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by writing your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. Which of the following would be the best response to

$$\sqrt{-1}\, 2^3 \sum \pi?$$

A and then my wife left me.
B but it was the wrong formula
C so we turned to the right
D but I thought the contest was today
E and it was delicious

2. A line with slope 3 passes through the point $(2, 3)$. What is its $y$-intercept?

A $-9$ B $-6$ C $-3$ D 3 E 9

3. If $a + b = 5$ and $a - b = 1$ what is $b$?

A 1 B 2 C 3 D 4 E 5

4. At the end of a certain trading day Stock XXX declined \$.27 or $4\%$ of its value. What was the value of the stock at the beginning of the trading day?

A \$6.75 B \$7.02 C \$6.48 D \$6.90 E \$7.10

5. What fraction is equivalent to the decimal $0.20192019\overline{2019}\ldots$?

A $\frac{673}{3333}$ B $\frac{2019}{10000}$ C $\frac{2018}{9999}$ D $\frac{2020}{9999}$ E $\frac{101}{5000}$

6. Suppose $f(x) = \sqrt{(x-3)^2}$. Find

$$\sum_{n=-2}^{4} f(n).$$

A $-14$ B $-10$ C 0 D 14 E 16

7. If $x > 0$ then

$$\sqrt[4]{x\sqrt[3]{x\sqrt{x}}}$$

simplifies to

A $\sqrt[8]{x}$ B $\sqrt[24]{x}$ C $\sqrt[8]{x^3}$ D $\sqrt[12]{x^7}$ E $x$

8. A line in the $(x, y)$ plane goes through the points $(-1, 3)$ and $(2, 15)$. Which of the following points does the line **not** go through.

A $(0, 7)$ B $(-2, -1)$ C $(\frac{1}{2}, 9)$ D $(1, 10)$ E $(2, 15)$

9. For all integers $n$, $(-1)^{3n^4+n+2}$ is equal to

A 1 B $n$ C $-1$ D $(-1)^n$ E none of these

10. The line given by the equation $y = 3x+2$ intersects the parabola $y = x^2$ at two points. Find the slope of the line segment that joins those two points.

A $\sqrt{17}$ B $-\sqrt{17}$ C 3 D $-3$ E $\frac{\sqrt{17}}{3}$

11. A math class has 40 students enrolled. The average on their final exam is 70. The 4 lowest scores were 12, 24, 26, and 38. What is the average of the other 36 students?

A 65 B 70 C 75 D 78 E 80

12. Suppose $N$ is a digit. What must $N$ be in order that $16616N19002321 = (2019)^4$?

A 1 B 4 C 5 D 7 E 8

13. How many three-digit numbers between 700 and 800 are there for which the hundreds digit equals the sum of the other two digits?

A 2 B 4 C 6 D 8 E none of these

14. What is the coefficient of $x^4$ if $(x + 2)^6$ were expanded?

A 4 B 15 C 30 D 60 E 120

15. The graph of the equation

$$x^2 - 8x + y^2 - 6y = 75$$

is a circle with what radius?

A 5 B $\sqrt{50}$ C 50 D $\sqrt{75}$ E 10

16. Suppose $x$ is a real number and

$$\sqrt{1-x} = 1 + x.$$

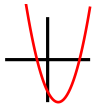
Which of the following statements is correct?

**A** There is no solution.
**B** There is exactly one solution; it is less than $-4$.
**C** There is exactly one solution; it is greater than $-4$.
**D** There are two solutions and one of them is less than $-4$.
**E** There are two solutions and one of them is greater than $-4$.

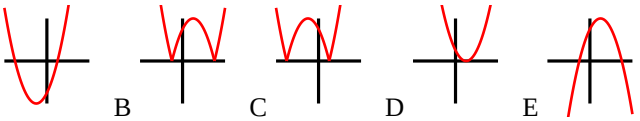
17. Which of the following statements about $n$ must be true if $x^2 + 6x + n > 13$, for all real numbers $x$?

A $n = 0$ B $n > 13$ C $n < 13$ D $n > 22$ E $n < -22$

18. Suppose $y = f(x)$ has the following graph:

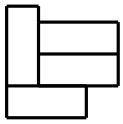


Which of the following is the graph of $y = |f(x)|$?



A B C D E

19. The figure below is made up of congruent rectangles that have perimeter 13. What is the perimeter of the figure?

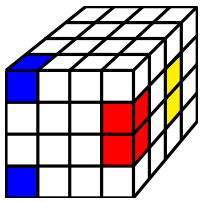


A 13 B 26 C 65 D 48 E cannot be determined

20. How may positive divisors does $8!$ have?

A 12 B 16 C 54 D 84 E 96

21. A $4 \times 4$ cube consists of 64 unit cubes. See the diagram below. What is the surface area of the structure obtained by removing the six unit cubes marked red, blue, and yellow.



A 64 B 76 C 96 D 104 E 128

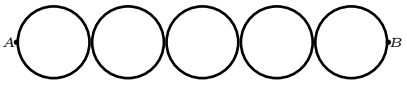
**Questions 22 - 28 Exact Answers**

*These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for $\pi$ or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or $6^{40}$.*

22. What is the sum of the roots to the polynomial

$$(x-2)(x+4) + (x-4)(x+3) + (x-6)(x+4)?$$

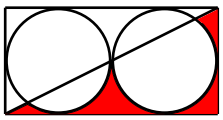
23. Five circles with the same radius are given in the diagram below. Their centers lie of the same line and neighboring circles are tangent. Let $A$ be the point on the left most circle and $B$ on the right most circles as indicated.



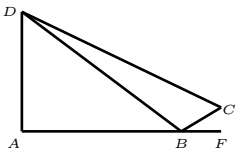

How many paths are there between $A$ and $B$ along the circles which do not meet the same point of tangency more than once?

24. Suppose $a$ and $b$ are integers greater than 2 such that $ab = 2019$. What is $a + b$?

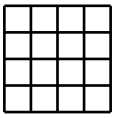
25. Two non overlapping circles of radius 2 sit in $4 \times 8$ rectangle. A diagonal is drawn. See the diagram below. Find the area of the region shaded red. That is, the area of the region in the triangle but outside of the circles.



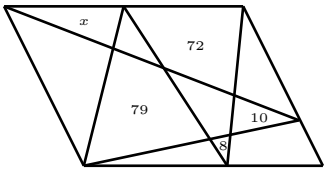
26. Find the area of quadrilateral $ABCD$ in the diagram below given that $A$, $B$, and $F$ are on the same line, $\angle DAB$ is a right angle, $\angle DBA = 25°$, $\angle CBF = 40°$, $\angle BDC = 5°$, and $|AB| = 10$.




27. How many rectangles can be formed from the grid lines in the following figure:



28. In the picture below $ABCD$ is a parallelogram with line segments that divide it up into various regions. The area of some of the regions are given. Find $x$.




**Tie Breaker requiring Full Solution**

Please give a **detailed explanation** of your solution to **Question 28**. Write your explanation on the **reverse side** of your answer sheet. *This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.*

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

**It is graded only to separate first, second, and third place ties.**