The Composite Cosine Transform on the Stiefel Manifold

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The λ-cosine transform on the unit sphere S^{n-1} in $\mathbb{R}^n$ is defined by

$$(T^\lambda f)(u) = \int\limits_{S^{n-1}} f(v)|v \cdot u|^\lambda \, dv, \qquad u \in S^{n-1}, \tag{0.1}$$

and has many applications. We introduce a new integral transform which generalizes $T^\lambda f$ for functions on the Stiefel/Grassmann manifold. We call it the composite cosine transform, by taking into account that its kernel agrees with the composite power function of the cone of positive definite symmetric matrices. The main concern is injectivity of the composite cosine transform and its particular cases. We study this problem by making use of the classical Fourier analysis on the space of rectangular matrices and the relevant zeta integrals. This approach differs from those of other authors and gives more complete results. In particular, we give an alternative proof of the result due to P. Goodey and R. Howard on non-injectivity of Matheron's transform (the case $\lambda = 1$) on the Grassmann manifold.

This is a joint work with Elena Ournycheva.