**A characterization of graphs with no octahedron minor**

**(Supplement)**

Several results in Section 7 are proved using a computer. In this note we explain how our proof works.

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1. **Computer programs**

Our computer-assisted proofs consist of only assertions of the following five types:

1. *L* is a minor of *J*
2. up to isomorphism, every partial addition of *H* is in {*J*1, *J*2, …, *J*k}
3. up to isomorphism, everyvertex split of *H* is in {*J*1, *J*2, …, *J*k}
4. up to isomorphism, every pentagonal extension of *H* is in {*J*1, *J*2, …, *J*k}
5. up to isomorphism, every hexagonal extension of *H* is in {*J*1, *J*2, …, *J*k}

We have five programs that verify these five types of assertions. For minor testing, we delete and contract the correct number of edges in all possible ways and then test for isomorphism; for assertions of type 2-5, we generate all extensions according to the corresponding definition and then test for membership.

In the following proofs, when an assertion of type 2-5 is made, we will simplify list graphs *J*1, *J*2, …, *J*k and we will not make any further justifications. Assertions of type 1 are made only when *L* is the octahedron or *V*10 (which only occurs four times). In every case, we also provide a set *X* of edges such that *J*/*X* has |*V*(*L*)| vertices and it contains *L* as a spanning subgraph. This extra information should help those who wish to verify some of the cases by hand.

To represent a split of a graph *H*, we list all its edges. For other extensions, we can represent the graph more efficiently. Since every partial addition of a graph *H* is obtained from *H* by adding one or two edges, when we list partial additions, we only need to list the extra edges in each partial addition. We will do the same when we list hexagonal extensions. Vertices of *H* will always be named 1, 2, …, *n*. In a hexagonal extension, the extra vertex will always be *n*+1. In a pentagonal extension *J*, the extra vertex will also be *n*+1. We will represent *J* by listing the three new edges {*n*+1, *u*}, {*n*+1, *v*}, {*n*+1, *w*}. Since there is exactly one edge *e* of *H* between *u, v,* and *w*, the three listed edges *e*1, *e*2, *e*3 uniquely determine *J*: *J* = *H*\*e* + *e*1 + *e*2 + *e*3*.*

1. **Case analysis**
2. *H* = *G*0914b = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 9}, {9, 8}, {8, 1}, {1, 5}, {2, 6}, {3, 7}, {4, 8}, {2, 9}}.

There are sixteen partial addition and all of them contain the octahedron:

* {{1, 3}} ; {{4, 5}, {6, 7}, {8, 9}}
* {{1, 4}} ; {{5, 6}, {8, 9}, {3, 7}}
* {{1, 6}} ; {{4, 5}, {8, 9}, {3, 7}}
* {{1, 7}} ; {{3, 4}, {5, 6}, {8, 9}}
* {{4, 6}} ; {{8, 9}, {1, 5}, {3, 7}}
* {{5, 9}} ; {{3, 4}, {6, 7}, {1, 8}}
* {{2, 4}, {3, 5}} ; {{5, 6}, {7, 9}, {1, 8}}
* {{2, 4}, {3, 6}} ; {{5, 6}, {7, 9}, {1, 8}}
* {{3, 5}, {4, 7}} ; {{5, 6}, {7, 9}, {1, 8}}
* {{3, 6}, {2, 7}} ; {{3, 4}, {8, 9}, {1, 5}}
* {{3, 6}, {4, 7}} ; {{5, 6}, {7, 9}, {1, 8}}
* {{3, 6}, {5, 7}} ; {{1, 2}, {7, 9}, {4, 8}}
* {{3, 6}, {7, 8}} ; {{3, 4}, {1, 5}, {2, 9}}
* {{5, 7}, {6, 9}} ; {{1, 2}, {3, 4}, {8, 9}}
* {{6, 9}, {2, 7}} ; {{3, 4}, {5, 6}, {1, 8}}
* {{6, 9}, {4, 7}} ; {{2, 3}, {5, 6}, {1, 8}}

Here in each of the 16 cases, the first part is the set of edges added to *H* to obtain the partial addition, and the second part is the set of edges to be contracted to obtain an octahedron minor. For instance, the first line says that (*H* + 13) / {45, 67, 89} contains the octahedron as a spanning subgraph while the last line says that (*H* + 47 + 69) / {23, 56, 18} contains the octahedron as a spanning subgraph.

There are two vertex splits, *G*1015a and *G*1015b.

There are three hexagonal extensions, one is *G*1017 and the other two contain the octahedron:

* {{1, 10}, {6, 10}, {9, 10}} ; {{4, 5}, {8, 9}, {3, 7}, {1, 10}}
* {{4, 10}, {6, 10}, {9, 10}} ; {{5, 6}, {1, 8}, {3, 7}, {4, 10}}

Here the first part lists the three edges added to obtain the hexagonal extensions and the second part lists the three edges that can be contracted to obtain an octahedron minor.

There are eight pentagonal extensions, four of which are *L*5’, G1016a, G1016b, G1016c, and the other four contain the octahedron:

* {{4, 10}, {1, 10}, {2, 10}} ; {{5, 6}, {8, 9}, {3, 7}, {1, 10}}
* {{4, 10}, {6, 10}, {7, 10}} ; {{8, 9}, {1, 5}, {3, 7}, {6, 10}}
* {{4, 10}, {2, 10}, {6, 10}} ; {{8, 9}, {1, 5}, {3, 7}, {6, 10}}
* {{6, 10}, {3, 10}, {4, 10}} ; {{8, 9}, {1, 5}, {3, 7}, {4, 10}}

Here the first part lists the three new edges and the second part lists the four edges that can be contracted to obtain an octahedron minor.

In the rest of the proofs, the same format will be used to report our results without further explanations.

1. *H* = *P*10 = {{1, 3}, {1, 4}, {2, 4}, {2, 5}, {3, 5}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {6, 10}, {1, 6}, {2, 7}, {3, 8}, {4, 9}, {5, 10}}

There is only one way to add an edge to *H*, which results in an octahedron minor.

* {{1, 2}} ; {{3, 5}, {7, 8}, {6, 10}, {4, 9}}

Since *H* is cubic, it has no splits.

1. *H* = *L*5’ = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 5}, {2, 6}, {3,7}, {4, 8}, {5, 9}, {6, 10}}

There are sixteen ways of adding an edge to *H* and all of them contain the octahedron:

* {{1, 3}} ; {{4, 5}, {7, 8}, {9, 10}, {2, 6}},
* {{1, 4}} ; {{1, 2}, {8, 9}, {9, 10}, {3, 7}},
* {{1, 6}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}},
* {{1, 7}} ; {{1, 2}, {3, 4}, {8, 9}, {9, 10}},
* {{1, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}},
* {{1, 9}} ; {{2, 3}, {4, 5}, {7, 8}, {6, 10}},
* {{2, 4}} ; {{1, 2}, {8, 9}, {9, 10}, {3, 7}},
* {{2, 5}} ; {{3, 4}, {6, 7}, {8, 9}, {1, 10}},
* {{2, 7}} ; {{1, 2}, {3, 4}, {8, 9}, {9, 10}},
* {{2, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}},
* {{2, 9}} ; {{2, 3}, {4, 5}, {7, 8}, {1, 10}},
* {{3, 5}} ; {{1, 2}, {6, 7}, {9, 10}, {4, 8}},
* {{3, 6}} ; {{1, 2}, {4, 5}, {7, 8}, {9, 10}},
* {{3, 8}} ; {{1, 2}, {4, 5}, {6, 7}, {9, 10}},
* {{4, 6}} ; {{1, 2}, {2, 3}, {7, 8}, {9, 10}},
* {{4, 7}} ; {{1, 2}, {2, 3}, {8, 9}, {9, 10}}

There are three splits, one is *L*5” and the other two contain the octahedron:

* {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {2, 6}, {3, 7}, {4, 8}, {5, 9}, {6, 10}, {1, 11}, {4, 11}, {5, 11}} ; {{2, 3}, {7, 8}, {1, 10}, {5, 9}, {4, 11}}
* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {2, 6}, {3, 7}, {4, 8}, {5, 9}, {6, 10}, {1, 11}, {6, 11}, {5, 11}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}, {1, 11}}

1. *H* = *G*1015a = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 7}, {2,8}, {3, 6}, {4, 9}, {5, 10}}

There are thirteen partial additions and all of them contain the octahedron:

* {{1, 4}} ; {{2, 3}, {5, 6}, {7, 8}, {9, 10}}
* {{1, 5}} ; {{3, 4}, {6, 7}, {9, 10}, {2, 8}}
* {{1, 9}} ; {{3, 4}, {6, 7}, {2, 8}, {5, 10}}
* {{3, 5}} ; {{6, 7}, {1, 10}, {2, 8}, {4, 9}}
* {{3, 9}} ; {{4, 5}, {6, 7}, {1, 10}, {2, 8}}
* {{3, 10}} ; {{1, 2}, {4, 5}, {6, 7}, {8, 9}}
* {{1, 3}, {2, 7}} ; {{3, 4}, {5, 6}, {8, 9}, {5, 10}}
* {{1, 3}, {2, 9}} ; {{3, 4}, {5, 6}, {7, 8}, {5, 10}}
* {{1, 6}, {2, 7}} ; {{2, 3}, {4, 5}, {8, 9}, {1, 10}}
* {{1, 6}, {7, 9}} ; {{2, 3}, {4, 5}, {1, 10}, {2, 8}}
* {{1, 8}, {2, 7}} ; {{2, 3}, {5, 6}, {1, 10}, {4, 9}}
* {{1, 3}, {2, 10}}; {{3, 4}, {5, 6}, {6, 7}, {8, 9}}
* {{1, 6}, {7, 10}}; {{1, 2}, {3, 4}, {8, 9}, {5, 10}}

Since *H* is cubic, it has no splits.

There are two hexagonal extensions and both contain the octahedron:

* {{1, 11}, {3, 11}, {5, 11}} ; {{3, 4}, {6, 7}, {9, 10}, {2, 8}, {1, 11}}
* {{1, 11}, {3, 11}, {9, 11}} ; {{3, 4}, {6, 7}, {2, 8}, {5, 10}, {1, 11}}

There are six pentagonal extensions, one is *G*1117 and other five contain the octahedron:

* {{1, 11}, {5, 11}, {6, 11}} ; {{3, 4}, {6, 7}, {9, 10}, {2, 8}, {5, 11}}
* {{1, 11}, {8, 11}, {9, 11}} ; {{3, 4}, {6, 7}, {2, 8}, {5, 10}, {9, 11}}
* {{3, 11}, {8, 11}, {9, 11}} ; {{4, 5}, {6, 7}, {1, 10}, {2, 8}, {9, 11}}
* {{3, 11}, {1, 11}, {7, 11}} ; {{5, 6}, {1, 10}, {2, 8}, {4, 9}, {7, 11}}
* {{4, 11}, {2, 11}, {8, 11}} ; {{2, 3}, {5, 6}, {9, 10}, {1, 7}, {8, 11}}

1. *H* = *G*1015b = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 7}, {2,6}, {3, 9}, {4, 8}, {5, 10}}

There are six partial additions and all of them contain the octahedron:

* {{1, 3}} ; {{4, 5}, {7, 8}, {9, 10}, {2, 6}}
* {{1, 4}} ; {{2, 3}, {5, 6}, {7, 8}, {9, 10}}
* {{1, 5}} ; {{2, 3}, {6, 7}, {9, 10}, {4, 8}}
* {{1, 6}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}}
* {{1, 9}} ; {{2, 3}, {6, 7}, {4, 8}, {5, 10}}
* {{2, 8}} ; {{3, 4}, {5, 6}, {9, 10}, {1, 7}}

Since *H* is cubic, it has no splits.

There are two hexagonal extensions and both contain the octahedron:

* {{1, 11}, {3, 11}, {5, 11}} ; {{2, 3}, {6, 7}, {9, 10}, {4, 8}, {1, 11}}
* {{1, 11}, {3, 11}, {8, 11}} ; {{2, 3}, {4, 5}, {6, 7}, {9, 10}, {1, 11}}

There are six pentagonal extensions, two of them are *G*1117 and *L*5”, and the other four contain the octahedron:

* {{1, 11}, {5, 11}, {6, 11}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}, {6, 11}}
* {{1, 11}, {8, 11}, {9, 11}} ; {{2, 3}, {4, 5}, {6, 7}, {9, 10}, {8, 11}}
* {{2, 11}, {4, 11}, {5, 11}} ; {{5, 6}, {7, 8}, {1, 10}, {3, 9}, {4, 11}}
* {{5, 11}, {1, 11}, {2, 11}} ; {{2, 3}, {6, 7}, {9, 10}, {4, 8}, {1, 11}}

1. *H* = *G*1016a = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 4}, {1, 6}, {2,7}, {2, 10}, {3, 9}, {5, 8}, {6, 10}}

There are eleven partial additions and all of them contain the octahedron:

* {{1, 3}} ; {{4, 5}, {6, 7}, {8, 9}, {9, 10}}
* {{1, 7}} ; {{2, 3}, {4, 5}, {8, 9}, {9, 10}}
* {{1, 8}} ; {{3, 4}, {4, 5}, {6, 7}, {9, 10}}
* {{2, 5}} ; {{3, 4}, {7, 8}, {9, 10}, {1, 4}}
* {{3, 5}} ; {{6, 7}, {8, 9}, {9, 10}, {1, 4}}
* {{3, 7}} ; {{1, 2}, {3, 4}, {9, 10}, {5, 8}}
* {{3, 8}} ; {{1, 2}, {4, 5}, {6, 7}, {9, 10}}
* {{4, 7}} ; {{1, 2}, {3, 4}, {9, 10}, {5, 8}}
* {{7, 9}} ; {{3, 4}, {9, 10}, {1, 6}, {5, 8}}
* {{3,10}} ; {{1, 2}, {4, 5}, {6, 7}, {8, 9}}
* {{7,10}} ; {{1, 2}, {4, 5}, {7, 8}, {3, 9}}

There are three splits, one is *G*1117 and the other two contain the octahedron:

* {{3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9,10}, {1, 4}, {1, 6}, {2, 7}, {2, 10}, {3, 9}, {5, 8}, {6, 10}, {1, 11}, {3,11}, {2, 11}} ; {{4, 5}, {7, 8}, {2, 10}, {3, 9}, {1, 11}}
* {{2,3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 4}, {1, 6}, {2, 7}, {3, 9}, {5, 8}, {6, 10}, {1, 11}, {10, 11}, {2, 11}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}, {1, 11}}

The only hexagonal extension is *G*1119a.

There are six pentagonal extensions, two of them are *G*1118a and *V*10+, and the other four contain the octahedron:

* {{3, 11}, {5, 11}, {8, 11}} ; {{1, 2}, {4, 5}, {6, 7}, {9, 10}, {8, 11}}
* {{8, 11}, {2, 11}, {3, 11}} ; {{1, 2}, {4, 5}, {6, 7}, {9, 10}, {3, 11}}
* {{8, 11}, {3, 11}, {4, 11}} ; {{1, 2}, {4, 5}, {6, 7}, {9, 10}, {3, 11}}
* {{8, 11}, {2, 11}, {10,11}} ; {{1, 2}, {4, 5}, {6, 7}, {3, 9}, {10, 11}}

1. *H* = *G*1016b = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 7}, {2,9}, {3, 8}, {4, 7}, {5, 10}, {6, 9}}

There are twelve partial additions and all of them contain the octahedron:

* {{1, 4}} ; {{1, 2}, {6, 7}, {3, 8}, {5, 10}}
* {{1, 6}} ; {{1, 2}, {3, 4}, {8, 9}, {5, 10}}
* {{2, 4}} ; {{1, 2}, {6, 7}, {3, 8}, {5, 10}}
* {{2, 5}} ; {{3, 4}, {5, 6}, {8, 9}, {1, 10}}
* {{2, 6}} ; {{1, 2}, {3, 4}, {8, 9}, {5, 10}}
* {{2, 7}} ; {{3, 4}, {5, 6}, {8, 9}, {1, 10}}
* {{2, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {5, 10}}
* {{3, 6}} ; {{1, 2}, {3, 4}, {7, 8}, {5, 10}}
* {{4, 6}} ; {{1, 2}, {2, 3}, {7, 8}, {5, 10}}
* {{4, 9}} ; {{1, 2}, {5, 6}, {1, 10}, {3, 8}}
* {{6, 8}} ; {{1, 2}, {2, 3}, {4, 5}, {9, 10}}
* {{2, 10}}; {{3, 4}, {5, 6}, {8, 9}, {1, 7}}}

There are five splits, two of them are *G*1117 and *L*5”, and the other three contain the octahedron:

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9,10}, {1, 10}, {2, 9}, {3, 8}, {5, 10}, {6, 9}, {1, 11}, {4, 11}, {7, 11}} ; {{1, 2}, {6, 7}, {3, 8}, {5, 10}, {4, 11}}
* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {2, 9}, {3, 8}, {4, 7}, {5, 10}, {6, 9}, {1, 11}, {6, 11}, {7, 11}} ; {{2, 3}, {4, 5}, {7, 8}, {1, 10}, {6, 11}}
* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 7}, {3, 8}, {4, 7}, {5, 10}, {2, 11}, {6, 11}, {9, 11}} ; {{1, 2}, {3, 4}, {8, 9}, {5, 10}, {6, 11}}

There are three hexagonal extensions, two of them are *G*1119a and *G*1119b, and the other contains the octahedron:

* {{2, 11}, {4, 11}, {10, 11}} ; {{1, 2}, {2, 3}, {5, 6}, {7, 8}, {4, 11}}

There are six pentagonal extensions, two of them are *G*1118a and *V*10+, and the other four contain the octahedron:

* {{2, 11}, {7, 11}, {8, 11}} ; {{1, 2}, {3, 4}, {6, 7}, {5, 10}, {8, 11}}
* {{2, 11}, {4, 11}, {7, 11}} ; {{1, 2}, {6, 7}, {3, 8}, {5, 10}, {4, 11}}
* {{7, 11}, {2, 11}, {3, 11}} ; {{3, 4}, {5, 6}, {8, 9}, {1, 10}, {2, 11}}
* {{7, 11}, {9, 11}, {10, 11}}; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {10, 11}}

1. *H* = *G*1016c = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 8}, {2,6}, {3,10}, {4, 9}, {5, 10}, {7, 10}}

There are seven partial additions and all of them contain the octahedron:

* {{1, 3}} ; {{4, 5}, {6, 7}, {8, 9}, {2, 6}}
* {{1, 4}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}}
* {{1, 5}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}}
* {{1, 6}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}}
* {{1, 7}} ; {{1, 2}, {3, 4}, {5, 6}, {8, 9}}
* {{1, 9}} ; {{1, 2}, {3, 4}, {5, 6}, {7, 8}}
* {{2, 9}} ; {{1, 2}, {3, 4}, {5, 6}, {7, 8}}

There are four splits, one is *G*1117 and the other three contain the octahedron:

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9,10}, {1, 8}, {2, 6}, {3, 10}, {4, 9}, {7, 10}, {1, 11}, {5, 11}, {10, 11}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}}
* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 8}, {2, 6}, {3, 10}, {4, 9}, {5, 10}, {1, 11}, {7, 11}, {10, 11}} ; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {7, 11}}
* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 8}, {2, 6}, {3, 10}, {4, 9}, {5, 10}, {7, 10}, {1, 11}, {9, 11}, {10, 11}} ; {{1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 11}}

There two hexagonal extensions, one is *G*1119a and the other contains the octahedron:

* {{1, 11}, {3, 11}, {9, 11}} ; {{1, 2}, {3, 4}, {5, 6}, {7, 8}, {1, 11}}

There are five pentagonal extensions, two of them are *G*1118a and *G*1118b, and the other three contain the octahedron:

* {{1, 11}, {6, 11}, {7, 11}} ; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {7, 11}}
* {{2, 11}, {4, 11}, {5, 11}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}}
* {{2, 11}, {5, 11}, {10, 11}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}}

1. *H* = *G1017* = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 1}, {1, 4}, {1, 6}, {2, 8}, {3,7}, {3, 10}, {5, 9}, {5, 10}, {8, 10}}

There are six partial addition and all of them contain the octahedron:

* {{2, 4}} ; {{4, 5}, {6, 7}, {8, 9}, {3, 10}}
* {{2, 6}} ; {{3, 4}, {6, 7}, {1, 9}, {5, 10}}
* {{2, 7}} ; {{3, 4}, {5, 6}, {8, 9}, {3, 10}}
* {{1, 10}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}}
* {{2, 10}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}}
* {{1, 7}, {3, 6}} ; {{1, 2}, {3, 4}, {1, 9}, {8, 10}}

There are two splits, and they both contain the octahedron:

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 6}, {2, 8}, {3, 7}, {3, 10}, {5, 9}, {5, 10}, {8, 10}, {2, 11}, {4, 11}, {1, 11}} ; {{4, 5}, {6, 7}, {1, 9}, {3, 10}, {2, 11}}
* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 4}, {2, 8}, {3, 7}, {3, 10}, {5, 9}, {5, 10}, {8, 10}, {2, 11}, {6, 11}, {1, 11}} ; {{3, 4}, {6, 7}, {1, 9}, {5, 10}, {2, 11}}

There is no hexagonal extension.

The only pentagonal extension is *G*1119a.

1. *H = L*5” = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6,7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {11, 1}, {1, 5}, {2, 10}, {3, 8}, {4, 7}, {5, 9}, {6, 11}}

There are twenty-one ways of adding an edge and all of them contain the octahedron:

* {{1, 3}} ; {{3, 4}, {6, 7}, {8, 9}, {2, 10}, {6, 11}}
* {{1, 4}} ; {{1, 2}, {6, 7}, {9, 10}, {10, 11}, {3, 8}}
* {{1, 6}} ; {{2, 3}, {3, 4}, {6, 7}, {8, 9}, {10, 11}}
* {{1, 7}} ; {{1, 2}, {3, 4}, {8, 9}, {9, 10}, {6, 11}}
* {{1, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {10, 11}}
* {{1, 9}} ; {{2, 3}, {3, 4}, {6, 7}, {7, 8}, {10, 11}}
* {{2, 5}} ; {{3, 4}, {6, 7}, {8, 9}, {9, 10}, {1, 11}}
* {{2, 6}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}, {1, 11}}
* {{2, 7}} ; {{1, 2}, {3, 4}, {8, 9}, {9, 10}, {6, 11}}
* {{2, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {10, 11}}
* {{2, 9}} ; {{3, 4}, {6, 7}, {8, 9}, {10, 11}, {1, 5}}
* {{5, 7}} ; {{1, 2}, {3, 4}, {8, 9}, {9, 10}, {6, 11}}
* {{5, 8}} ; {{1, 2}, {2, 3}, {9, 10}, {4, 7}, {6, 11}}
* {{6, 8}} ; {{1, 2}, {2, 3}, {9, 10}, {10, 11}, {4, 7}}
* {{6, 9}} ; {{2, 3}, {4, 5}, {7, 8}, {9, 10}, {1, 11}}
* {{7, 9}} ; {{1, 2}, {4, 5}, {9, 10}, {3, 8}, {6, 11}}
* {{1, 10}}; {{2, 3}, {3, 4}, {6, 7}, {8, 9}, {6, 11}}
* {{2, 11}}; {{3, 4}, {6, 7}, {8, 9}, {9, 10}, {1, 5}}
* {{7, 10}}; {{1, 2}, {2, 3}, {4, 5}, {8, 9}, {6, 11}}
* {{7, 11}}; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {9, 10}}
* {{8, 10}}; {{1, 2}, {2, 3}, {4, 7}, {5, 9}, {6, 11}}

There are two splits and they both contain the octahedron:

* {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 11}, {2, 10}, {3, 8}, {4,7}, {5, 9}, {6, 11}, {1, 12}, {4, 12}, {5, 12}} ; {{2, 3}, {5, 6}, {7, 8}, {9, 10}, {1, 11}, {4, 12}}
* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 11}, {2, 10}, {3, 8}, {4,7}, {5, 9}, {6, 11}, {1, 12}, {6, 12}, {5, 12}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}, {1, 12}}

1. *H* = *G*1117 = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {11, 1}, {1, 4}, {2, 9}, {3, 6}, {4, 8}, {5, 11}, {7, 10}}

There are twenty partial additions and all of them contain the octahedron:

* {{1, 3}} ; {{4, 5}, {6, 7}, {7, 8}, {10, 11}, {2, 9}}
* {{1, 5}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {10, 11}}
* {{1, 6}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}}
* {{1, 7}} ; {{1, 2}, {8, 9}, {10, 11}, {3, 6}, {5, 11}}
* {{1, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {5, 11}}
* {{1, 9}} ; {{2, 3}, {4, 5}, {6, 7}, {7, 8}, {10, 11}}
* {{2, 7}} ; {{1, 2}, {8, 9}, {10, 11}, {3, 6}, {5, 11}}
* {{2, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {5, 11}}
* {{3, 7}} ; {{1, 2}, {5, 6}, {9, 10}, {4, 8}, {5, 11}}
* {{3, 8}} ; {{1, 2}, {4, 5}, {6, 7}, {8, 9}, {10, 11}}
* {{4, 7}} ; {{1, 2}, {2, 3}, {5, 6}, {8, 9}, {10, 11}}
* {{7, 9}} ; {{1, 2}, {2, 3}, {5, 6}, {10, 11}, {4, 8}}
* {{1, 10}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}}
* {{2, 10}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}}
* {{3, 10}}; {{1, 2}, {4, 5}, {6, 7}, {8, 9}, {10, 11}}
* {{3, 11}}; {{1, 2}, {4, 5}, {6, 7}, {8, 9}, {10, 11}}
* {{4, 10}}; {{1, 2}, {2, 3}, {6, 7}, {8, 9}, {5, 11}}
* {{7, 11}}; {{1, 2}, {2, 3}, {5, 6}, {9, 10}, {4, 8}}
* {{8, 10}}; {{1, 2}, {3, 4}, {6, 7}, {2, 9}, {5, 11}}
* {{8, 11}}; {{1, 2}, {2, 3}, {4, 5}, {6, 7}, {9, 10}}

There are two splits, and they both contain the octahedron:

* {{1, 2}, {2, 3}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 11}, {2, 9}, {3, 6}, {4, 8}, {5, 11}, {7, 10}, {1, 12}, {3, 12}, {4, 12}} ; {{4, 5}, {7, 8}, {10, 11}, {2, 9}, {3, 6}, {1, 12}}
* {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 11}, {2, 9}, {3, 6}, {4, 8}, {5, 11}, {7, 10}, {1, 12}, {5, 12}, {4, 12}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {10, 11}, {5, 12}}

There are six hexagonal extensions, one is *G*1220 and the other five contain the octahedron:

* {{1, 12}, {3, 12}, {5, 12}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {10, 11}, {1, 12}}
* {{2, 12}, {6, 12}, {8, 12}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {5, 11}, {2, 12}}
* {{3, 12}, {7, 12}, {9, 12}} ; {{1, 2}, {2, 3}, {5, 6}, {10, 11}, {4, 8}, {7, 12}}
* {{2, 12}, {6, 12}, {10, 12}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}, {2, 12}}
* {{4, 12}, {7, 12}, {11, 12}}; {{1, 2}, {2, 3}, {5, 6}, {8, 9}, {9, 10}, {7, 12}}

There are nine pentagonal extensions and all of them contain the octahedron:

* {{1, 12}, {9, 12}, {10, 12}} ; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}, {10, 12}}
* {{2, 12}, {4, 12}, {8, 12}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {5, 11}, {8, 12}}
* {{4, 12}, {6, 12}, {7, 12}} ; {{1, 2}, {2, 3}, {5, 6}, {8,9}, {10, 11}, {7, 12}}
* {{7, 12}, {3, 12}, {4, 12}} ; {{1, 2}, {2, 3}, {5, 6}, {8, 9}, {10, 11}, {7, 12}}
* {{7, 12}, {4, 12}, {5, 12}} ; {{1, 2}, {2, 3}, {5, 6}, {8, 9}, {10, 11}, {4, 12}}
* {{7, 12}, {5, 12}, {11, 12}} ; {{1, 2}, {2, 3}, {5, 6}, {9, 10}, {4, 8}, {11, 12}}
* {{2, 12}, {10, 12}, {11, 12}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {5, 11}, {10, 12}}
* {{10, 12}, {1, 12}, {2, 12}} ; {{2, 3}, {5, 6}, {6, 7}, {8, 9}, {1, 11}, {2, 12}}
* {{11, 12}, {6, 12}, {7, 12}} ; {{1, 2}, {2, 3}, {5, 6}, {9, 10}, {4, 8}, {7, 12}}

1. *H* = *G*1118a = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 6}, {1, 10}, {2, 9}, {3, 10}, {4, 7}, {5, 10}, {6, 11}, {8, 11}}

There are sixteen partial additions and all of them contain the octahedron:

* {{1, 3}} ; {{3, 4}, {4, 5}, {7, 8}, {10, 11}, {2, 9}}
* {{1, 4}} ; {{2, 3}, {4, 5}, {7, 8}, {8, 9}, {10, 11}}
* {{1, 5}} ; {{1, 2}, {2, 3}, {8, 9}, {4, 7}, {6, 11}}
* {{1, 7}} ; {{1, 2}, {2, 3}, {4, 5}, {8, 9}, {6, 11}}
* {{2, 5}} ; {{1, 2}, {2, 3}, {8, 9}, {4, 7}, {6, 11}}
* {{2, 7}} ; {{1, 2}, {2, 3}, {4, 5}, {8, 9}, {6, 11}}
* {{3, 5}} ; {{1, 2}, {2, 3}, {8, 9}, {4, 7}, {6, 11}}
* {{3, 6}} ; {{1, 2}, {4, 5}, {7, 8}, {8, 9}, {10, 11}}
* {{3, 7}} ; {{1, 2}, {2, 3}, {4, 5}, {8, 9}, {6, 11}}
* {{3, 8}} ; {{1, 2}, {4, 5}, {7, 8}, {8, 9}, {10, 11}}
* {{3, 9}} ; {{1, 2}, {3, 4}, {4, 5}, {7, 8}, {10, 11}}
* {{5, 7}} ; {{1, 2}, {2, 3}, {3, 4}, {8, 9}, {6, 11}}
* {{3, 11}}; {{1, 2}, {2, 3}, {4, 5}, {7, 8}, {8, 9}}
* {{4, 11}}; {{1, 2}, {2, 3}, {4, 5}, {6, 7}, {8, 9}}
* {{5, 11}}; {{1, 2}, {2, 3}, {3, 4}, {6, 7}, {8, 9}}
* {{7, 10}}; {{1, 2}, {2, 3}, {4, 5}, {8, 9}, {6, 11}}

There are eight splits and all of them contain the octahedron:

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 10}, {2, 9}, {3, 10}, {4, 7}, {5, 10}, {6, 11}, {8, 11}, {1, 12}, {5, 12}, {6, 12}};

{{1, 2}, {2, 3}, {8, 9}, {4, 7}, {6, 11}, {5, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 10}, {2, 9}, {3, 10}, {4, 7}, {5, 10}, {6, 11}, {8, 11}, {1, 12}, {7, 12}, {6, 12}};

{{1, 2}, {2, 3}, {4, 5}, {8, 9}, {6, 11}, {7, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 6}, {2, 9}, {4, 7}, {5, 10}, {6, 11}, {8, 11}, {1, 12}, {3, 12}, {10, 12}};

{{3, 4}, {4, 5}, {7, 8}, {10, 11}, {2, 9}, {1, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 6}, {2, 9}, {3, 10}, {4, 7}, {6, 11}, {8, 11}, {1, 12}, {5, 12}, {10, 12}};

{{1, 2}, {2, 3}, {8, 9}, {4, 7}, {6, 11}, {5, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 6}, {1, 10}, {2, 9}, {4, 7}, {6, 11}, {8, 11}, {3, 12}, {5, 12}, {10, 12}};

{{1, 2}, {2, 3}, {8, 9}, {4, 7}, {6, 11}, {5, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {10,11}, {1, 6}, {1, 10}, {2, 9}, {4, 7}, {5, 10}, {6, 11}, {8, 11}, {3, 12}, {9, 12}, {10, 12}};

{{1, 2}, {3, 4}, {4, 5}, {7, 8}, {10, 11}, {9, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 6}, {1, 10}, {2, 9}, {4, 7}, {5, 10}, {6, 11}, {8, 11}, {3, 12}, {11, 12}, {10, 12}};

{{1, 2}, {2, 3}, {4, 5}, {7, 8}, {8, 9}, {11, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 6}, {1, 10}, {2, 9}, {3, 10}, {4, 7}, {6, 11}, {8, 11}, {5, 12}, {11, 12}, {10, 12}};

{{1, 2}, {3, 4}, {4, 5}, {6, 7}, {8, 9}, {11, 12}}

There are four hexagonal extensions, one is *G*1221, the next contains the octahedron, and the last two contain *V*10:

* {{3, 12}, {7, 12}, {9, 12}} ; {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {8, 11}, {7, 12}}
* {{2, 12}, {4, 12}, {6, 12}} ; {{1, 2}, {4, 5}}
* {{2, 12}, {4, 12}, {8, 12}} ; {{2, 3}, {10, 11}}

Since *H* is bipartite, it has no pentagonal extension.

1. *H* = *G*1118b = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 8}, {2,9}, {3,11}, {4, 9}, {5, 10}, {6, 9}, {7, 11}, {9, 11}}

There are three partial additions and all of them contain the octahedron:

* {{1, 4}} ; {{1, 2}, {5, 6}, {7, 8}, {1, 10}, {3, 11}}
* {{2, 4}} ; {{1, 2}, {5, 6}, {7, 8}, {1, 10}, {3, 11}}
* {{2, 6}} ; {{1, 2}, {3, 4}, {7, 8}, {3, 11}, {5, 10}}

There are four splits and all of them contain the octahedron:

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 8}, {3, 11}, {5, 10}, {6, 9}, {7, 11}, {9, 11}, {2, 12}, {4, 12}, {9, 12}};

{{1, 2}, {5, 6}, {7, 8}, {1, 10}, {3, 11}, {4, 12}},

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 8}, {3, 11}, {4, 9}, {5, 10}, {7, 11}, {9, 11}, {2, 12}, {6, 12}, {9, 12}};

{{1, 2}, {3, 4}, {7, 8}, {3, 11}, {5, 10}, {6, 12}},

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 8}, {3, 11}, {5, 10}, {7, 11}, {9, 11}, {2, 12}, {4, 12}, {6, 12}, {9, 12}};

{{1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 10}, {3, 11}},

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 10}, {1, 8}, {3, 11}, {5, 10}, {6, 9}, {7, 11}, {9, 11}, {2, 12}, {4, 12}, {10, 12}, {9, 12}};
* {{1, 2}, {3, 4}, {5, 6}, {7, 8}, {3, 11}, {5, 10}}

There are two hexagonal extensions, one is *G*1221 and the other contains the octahedron:

* {{2, 12}, {4, 12}, {10, 12}} ; {{1, 2}, {3, 4}, {5, 6}, {7, 8}, {3, 11}, {4, 12}}

Since *H* is bipartite, it has no pentagonal extension.

1. *H* = *G*1119a = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 4}, {1, 6}, {2,9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9, 11}}

There are seventeen partial additions and all of them contain the octahedron:

* {{1, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {9, 10}, {3, 11}}
* {{2, 4}} ; {{4, 5}, {6, 7}, {7, 8}, {9, 10}, {3, 11}}
* {{2, 5}} ; {{3, 4}, {6, 7}, {7, 8}, {9, 10}, {3, 11}}
* {{2, 6}} ; {{2, 3}, {3, 4}, {7, 8}, {1, 10}, {5, 11}}
* {{2, 7}} ; {{3, 4}, {5, 6}, {7, 8}, {9, 10}, {3, 11}}
* {{3, 6}} ; {{1, 2}, {3, 4}, {7, 8}, {1, 10}, {9, 11}}
* {{4, 6}} ; {{1, 2}, {3, 4}, {7, 8}, {1, 10}, {9, 11}}
* {{4, 7}} ; {{1, 2}, {5, 6}, {7, 8}, {9, 10}, {3, 11}}
* {{4, 8}} ; {{1, 2}, {3, 4}, {6, 7}, {1, 10}, {9, 11}}
* {{6, 8}} ; {{1, 2}, {3, 4}, {1, 10}, {3, 7}, {9, 11}}
* {{1, 11}}; {{1, 2}, {3, 4}, {6, 7}, {7, 8}, {9, 10}}
* {{2, 10}}; {{2, 3}, {3, 4}, {7, 8}, {1, 6}, {5, 11}}
* {{3, 10}}; {{1, 2}, {3, 4}, {6, 7}, {7, 8}, {5, 11}}
* {{4, 10}}; {{1, 2}, {3, 4}, {6, 7}, {7, 8}, {5, 11}}
* {{4, 11}}; {{1, 2}, {5, 6}, {7, 8}, {8, 9}, {9, 10}}
* {{6, 10}}; {{1, 2}, {2, 3}, {3, 4}, {7, 8}, {5, 11}}
* {{7, 10}}; {{1, 2}, {3, 4}, {5, 6}, {7, 8}, {3, 11}}

There are eleven splits, one is *G*1220 and the other ten contain the octahedron:

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9, 11}, {2, 12}, {4, 12}, {1, 12}};

{{4, 5}, {6, 7}, {7, 8}, {1, 10}, {3, 11}, {2, 12}}

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9, 11}, {2, 12}, {6, 12}, {1, 12}};

{{2, 3}, {3, 4}, {7, 8}, {1, 10}, {5, 11}, {6, 12}}

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9,11}, {2, 12}, {10, 12}, {1, 12}};

{{2, 3}, {3, 4}, {7, 8}, {1, 6}, {5, 11}, {10, 12}}

* {{1, 2}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9, 11}, {2, 12}, {4, 12}, {3, 12}};

{{4, 5}, {6, 7}, {7, 8}, {9, 10}, {3, 11}, {2, 12}}

* {{1, 2}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {2, 9}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9, 11}, {2, 12}, {7, 12}, {3, 12}};

{{3, 4}, {6, 7}, {7, 8}, {1, 10}, {9, 11}, {2, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {5, 11}, {9, 11}, {4, 12}, {6, 12}, {5, 12}};

{{1, 2}, {3, 4}, {7, 8}, {1, 10}, {9, 11}, {6, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 10}, {5, 11}, {9, 11}, {4, 12}, {8, 12}, {5, 12}};

{{1, 2}, {3, 4}, {6, 7}, {1, 10}, {9, 11}, {8, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 11}, {9,11}, {4, 12}, {10, 12}, {5, 12}};

{{1, 2}, {3, 4}, {6, 7}, {7, 8}, {5, 11}, {10, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 10}, {9,11}, {4, 12}, {11, 12}, {5, 12}};

{{1, 2}, {6, 7}, {7, 8}, {5, 10}, {9, 11}, {4, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {2, 9}, {3, 7}, {3, 11}, {5, 8}, {5, 11}, {9,11}, {6, 12}, {10, 12}, {5, 12}};

{{1, 2}, {3, 4}, {6, 7}, {7, 8}, {9, 11}, {10, 12}}

The only hexagonal extension is *G*1222.

There are four pentagonal extensions, one is *G*1221, one contains *V*10, and two contain the octahedron:

* {{1, 12}, {3, 12}, {7, 12}} ; {{3, 4}, {9, 10}}
* {{3, 12}, {5, 12}, {6, 12}} ; {{1, 2}, {3, 4}, {7, 8}, {1, 10}, {9, 11}, {6, 12}}
* {{3, 12}, {8, 12}, {9, 12}} ; {{1, 2}, {3, 4}, {6, 7}, {1, 10}, {9, 11}, {8, 12}}

1. *H* = *G*1119b = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 1}, {1, 6}, {1, 11}, {2, 7}, {3, 9}, {3, 10}, {4, 7}, {5, 10}, {5, 11}, {8, 10}, {8, 11}}

There are five partial additions and all of them contain the octahedron:

* {{1, 4}} ; {{1, 2}, {5, 6}, {8, 9}, {1, 11}, {5, 10}}
* {{2, 4}} ; {{1, 2}, {5, 6}, {8, 9}, {1, 11}, {5, 10}}
* {{2, 8}} ; {{3, 4}, {5, 6}, {1, 9}, {1, 11}, {5, 10}}
* {{2, 9}} ; {{3, 4}, {4, 5}, {6, 7}, {1, 11}, {3, 10}}
* {{2, 10}}; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {1, 11}}

There are four splits and all of them contain the octahedron:

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1,11}, {2, 7}, {3, 9}, {3, 10}, {4, 7}, {5, 10}, {5, 11}, {8, 10}, {8, 11}, {2, 12}, {6, 12}, {1, 12}};

{{2, 3}, {4, 5}, {1, 9}, {1, 11}, {8, 10}, {6, 12}}

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 6}, {1, 11}, {2, 7}, {3, 9}, {3, 10}, {4, 7}, {5, 10}, {5, 11}, {8, 10}, {8, 11}, {2, 12}, {9, 12}, {1, 12}};

{{3, 4}, {6, 7}, {8, 9}, {1, 11}, {5, 10}, {2, 12}}

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 6}, {2, 7}, {3, 9}, {3, 10}, {4, 7}, {5, 10}, {5, 11}, {8, 10}, {8, 11}, {2, 12}, {11, 12}, {1, 12}};

{{2, 3}, {3, 4}, {6, 7}, {1, 9}, {5, 10}, {11, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 6}, {1, 11}, {3, 9}, {3, 10}, {5, 10}, {5, 11}, {8, 10}, {8, 11}, {2, 12}, {4, 12}, {7, 12}};

{{1, 2}, {6, 7}, {8, 9}, {1, 11}, {5, 10}, {4, 12}}

The only hexagonal extension is *G*1222.

There is only one pentagonal extension, which contains *V*10:

* {{1, 12}, {7, 12}, {8, 12}} ; {{1, 2}, {1, 11}}

1. *H* = *G*1220 = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {11, 1}, {1, 9}, {2, 5}, {2, 7}, {3, 8}, {4, 10}, {4, 12}, {6, 11}, {6, 12}, {8, 12}}

There are nine ways of adding edges and all of them contain the octahedron:

* {{1, 3}} ; {{3, 4}, {4, 5}, {8, 9}, {10, 11}, {2, 7}, {6, 12}}
* {{1, 4}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}, {4, 12}}
* {{1, 6}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}, {4, 12}}
* {{1, 7}} ; {{2, 3}, {3, 4}, {4, 5}, {8, 9}, {10, 11}, {6, 12}}
* {{3, 5}} ; {{1, 2}, {5, 6}, {6, 7}, {8, 9}, {10, 11}, {4, 12}}
* {{3, 7}} ; {{1, 2}, {3, 4}, {4, 5}, {8, 9}, {10, 11}, {6, 12}}
* {{1, 10}}; {{2, 3}, {4, 5}, {6, 7}, {8, 9}, {4, 12}, {6, 11}}
* {{1, 12}}; {{1, 2}, {2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}}
* {{2, 12}}; {{1, 2}, {2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}}

There are two splits, and both contain the octahedron:

* {{3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 11}, {1, 9}, {2, 5}, {2, 7}, {3, 8}, {4, 10}, {4, 12}, {6, 11}, {6, 12}, {8, 12}, {1, 13}, {3, 13}, {2, 13}};

{{3, 4}, {4, 5}, {8, 9}, {10, 11}, {2, 7}, {6, 12}, {1, 13}}

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 11}, {1, 11}, {1, 9}, {2, 5}, {3, 8}, {4, 10}, {4, 12}, {6, 11}, {6, 12}, {8, 12}, {1, 13}, {7, 13}, {2, 13}};

{{2, 3}, {5, 6}, {6, 7}, {8, 9}, {10, 11}, {4, 12}, {1, 13}}

The only hexagonal extension contains the octahedron:

* {{1, 13}, {6, 13}, {8, 13}} ; {{2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}, {4, 12}, {1, 13}}

The only pentagonal extension contains the octahedron:

* {{2, 13}, {8, 13}, {9, 13}} ; {{2, 3}, {4, 5}, {6, 7}, {1, 11}, {4, 10}, {4, 12}, {9, 13}}

1. *H* = *G*1221 = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {10, 1}, {1, 4}, {1,6}, {2,7}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {7, 12}, {9, 12}}

There are eleven partial additions and all of them contain the octahedron:

* {{1, 8}} ; {{1, 2}, {3, 4}, {5, 6}, {9, 10}, {3, 11}, {5, 12}}
* {{2, 4}} ; {{4, 5}, {5, 6}, {8, 9}, {1, 10}, {3, 11}, {5, 12}}
* {{2, 6}} ; {{2, 3}, {3, 4}, {8, 9}, {1, 10}, {3, 11}, {5, 12}}
* {{2, 8}} ; {{1, 2}, {3, 4}, {5, 6}, {9, 10}, {3, 11}, {5, 12}}
* {{2, 9}} ; {{3, 4}, {5, 6}, {7, 8}, {9, 10}, {3, 11}, {5, 12}}
* {{4, 7}} ; {{1, 2}, {5, 6}, {8, 9}, {9, 10}, {3, 11}, {5, 12}}
* {{4, 8}} ; {{1, 2}, {4, 5}, {5, 6}, {9, 10}, {3, 11}, {5, 12}}
* {{4, 9}} ; {{1, 2}, {5, 6}, {7, 8}, {9, 10}, {3, 11}, {5, 12}}
* {{1, 11}}; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {9, 10}, {5, 12}}
* {{2, 12}}; {{1, 2}, {3, 4}, {5, 6}, {8, 9}, {9, 10}, {3, 11}}
* {{8, 10}}; {{1, 2}, {3, 4}, {5, 6}, {1, 10}, {3, 11}, {9, 12}}

There are nine splits and they all contain the octahedron:

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 6}, {2, 7}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {7, 12}, {9, 12}, {2, 13}, {4, 13}, {1, 13}};

{{4, 5}, {5, 6}, {8, 9}, {1, 10}, {3, 11}, {5, 12}, {2, 13}}

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {2, 7}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {7, 12}, {9, 12}, {2, 13}, {6, 13}, {1, 13}};

{{2, 3}, {3, 4}, {8, 9}, {1, 10}, {3, 11}, {5, 12}, {6, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {7, 12}, {9, 12}, {2, 13}, {6, 13}, {7, 13}};

{{2, 3}, {3, 4}, {8, 9}, {1, 10}, {3, 11}, {5, 12}, {6, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {7, 12}, {9, 12}, {2, 13}, {8, 13}, {7, 13}};

{{1, 2}, {3, 4}, {5, 6}, {9, 10}, {3, 11}, {5, 12}, {8, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 10}, {1, 10}, {1, 4}, {1, 6}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {9, 12}, {2, 13}, {12, 13}, {7, 13}};

{{1, 2}, {3, 4}, {5, 6}, {8, 9}, {9, 10}, {3, 11}, {12, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {2, 7}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 11}, {7, 12}, {9, 12}, {8, 13}, {10, 13}, {7, 13}};

{{1, 2}, {3, 4}, {5, 6}, {1, 10}, {3, 11}, {9, 12}, {8, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {8, 9}, {9, 10}, {1, 10}, {1, 4}, {1, 6}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 11}, {7, 12}, {9, 12}, {2, 13}, {6, 13}, {8, 13}, {7, 13}};

{{1, 2}, {3, 4}, {5, 6}, {8, 9}, {9, 10}, {3, 11}, {5, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 11}, {7, 12}, {9, 12}, {2, 13}, {6, 13}, {10, 13}, {7, 13}};

{{1, 2}, {3, 4}, {5, 6}, {8, 9}, {9, 10}, {3, 11}, {5, 12}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {7, 8}, {8, 9}, {9, 10}, {1,10}, {1, 4}, {1, 6}, {3, 8}, {3, 11}, {5, 11}, {5, 12}, {7, 10}, {7, 12}, {9, 12}, {2, 13}, {6, 13}, {11, 13}, {7, 13}};

{{1, 2}, {3, 4}, {5, 6}, {8, 9}, {1, 10}, {3, 11}, {5, 12}}

The only hexagonal extension contains *V*10.

* {{1, 13}, {3, 13}, {9, 13}} ; {{1, 2}, {5, 6}, {7, 8}}

There is no pentagonal extension.

1. *H* = *G*1222 = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {9, 1}, {1, 4}, {1, 7}, {1,12}, {2, 8}, {3, 10}, {3, 11}, {3, 12}, {5, 9}, {5, 10}, {6, 11}, {6, 12}, {8, 10}, {8, 11}}

There are seven partial additions and all of them contain the octahedron:

* {{2, 4}} ; {{4, 5}, {6, 7}, {8, 9}, {1, 12}, {3, 10}, {3, 11}}
* {{2, 5}} ; {{3, 4}, {6, 7}, {8, 9}, {1, 12}, {3, 10}, {3, 11}}
* {{4, 6}} ; {{1, 2}, {6, 7}, {8, 9}, {1, 12}, {3, 10}, {3, 11}}
* {{4, 7}} ; {{1, 2}, {3, 4}, {1, 9}, {1, 12}, {3, 11}, {5, 10}}
* {{4, 9}} ; {{1, 2}, {3, 4}, {6, 7}, {1, 12}, {3, 11}, {5, 10}}
* {{1, 10}}; {{1, 2}, {3, 4}, {6, 7}, {8, 9}, {1, 12}, {3, 11}}
* {{4, 12}}; {{1, 2}, {4, 5}, {6, 7}, {8, 9}, {3, 10}, {3, 11}}

There are five splits and all of them contain the octahedron:

* {{2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 7}, {1, 12}, {2, 8}, {3, 10}, {3, 11}, {3, 12}, {5, 9}, {5, 10}, {6, 11}, {6, 12}, {8, 10}, {8, 11}, {2, 13}, {4, 13}, {1, 13}};

{{4, 5}, {6, 7}, {8, 9}, {1, 12}, {3, 10}, {3, 11}, {2, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 12}, {2, 8}, {3, 10}, {3, 11}, {3, 12}, {5, 9}, {5, 10}, {6, 11}, {6, 12}, {8, 10}, {8, 11}, {4, 13}, {7, 13}, {1, 13}};

{{1, 2}, {3, 4}, {1, 9}, {1, 12}, {3, 11}, {5, 10}, {7, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 7}, {1, 12}, {2, 8}, {3, 10}, {3, 11}, {3, 12}, {5, 9}, {5, 10}, {6, 11}, {6, 12}, {8, 10}, {8, 11}, {4, 13}, {9, 13}, {1, 13}};

{{1, 2}, {3, 4}, {6, 7}, {1, 12}, {3, 11}, {5, 10}, {9, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 7}, {2, 8}, {3, 10}, {3, 11}, {3, 12}, {5, 9}, {5, 10}, {6,11}, {6, 12}, {8, 10}, {8, 11}, {4, 13}, {12, 13}, {1, 13}};

{{1, 2}, {4, 5}, {6, 7}, {8, 9}, {3, 10}, {3, 11}, {12, 13}}

* {{1, 2}, {2, 3}, {3, 4}, {6, 7}, {7, 8}, {8, 9}, {1, 9}, {1, 4}, {1, 7}, {1, 12}, {2, 8}, {3, 10}, {3, 11}, {3, 12}, {5, 9}, {5, 10}, {6, 11}, {6, 12}, {8, 10}, {8, 11}, {4, 13}, {6, 13}, {5, 13}};

{{1, 2}, {6, 7}, {1, 9}, {1, 12}, {3, 11}, {5, 10}, {4, 13}}

There is no hexagonal extension.

There only pentagonal extension contains *V*10:

* {{1, 13}, {5, 13}, {6, 13}} ; {{1, 2}, {3, 4}, {6, 7}}