```
<< KnotTheory`
```

```
Loading KnotTheory`...
```

```
L[x_] := Module[{xx}, (Do[
     If[ x[[i, 1]] == 1, xx = x[[i, 3]] ];
     If[ x[[i, 2]] == 1, xx = x[[i, 4]] ];
     If[ x[[i, 3]] == 1, xx = x[[i, 1]]];
     If[ x[[i, 4]] == 1, xx = x[[i, 2]] ]
     , {i, 2, Length[x]}]; xx)]
```

L Finds the length of the first component

```
L1[PD[Xs___X]] := (le = L[PD[Xs]];  PD[Xs] /.
    {X[i_, j_, k_, l_] /; i > le && j > le :> 1, X[i_, j_, k_, l_] /; i ≤ le && j > le :> P[i, k],
     X[i_, j_, k_, l_] /; i > le && j <= le :> P[j, l]})
```

L1 gives the first component, writes first component in X's P's and 1's (1 represents the empty link and works in the following command):

```
KB[pd_] := Expand[
  Expand[Times @@ pd /. X[a_, b_, c_, d_] :> 1 / A P[a, d] P[b, c] + A P[a, b] P[c, d]] //.
   {P[a_, b_] P[b_, c_] :> P[a, c], P[a_, b_]^2 :> P[a, a], P[a_, a_] → -A^2 - 1 / A^2}]
```

Bar-Natan's command for Kauffman bracket, modified so that it gives normal bracket (rather than the conjugate.)

```
L2[PD[Xs___X]] :=
  (le = L[PD[Xs]];  PD[Xs] /.
    {X[i_, j_, k_, l_] /; i <= le && j <= le :> 1, X[i_, j_, k_, l_] /; i > le && j <= le :> P[i, k],
     X[i_, j_, k_, l_] /; i <= le && j > le :> P[j, l]})
```

L2 gives the first component, writes first component in X's P's and 1's (1 represents the empty link and works in the following command):

```
count[x_] :=
   (j = 0; Do[
     If[ x[[ i]] == 1 || x[[ i]] == -1, j += x[[i] ]] ,
     {i, 1, Length[x]}
    ]; j);
```

```
General::spell1 :
 Possible spelling error: new symbol name "count" is similar to existing symbol "Count". More…
```

```
w[PD[Xs___X]] :=
  (le = L[PD[Xs]];  count[
    PD[Xs] /. {X[i_, j_, k_, l_] /; j == l + 1 || (l ==  le && j == 1) || ( j == le + 1 && l  != le + 2)
         :> 1,
       X[i_, j_, k_, l_] /; l == j + 1 || (j ==  le && l == 1)  ||  (l == le + 1  &&  j  != le + 2)
         :> - 1}])
```

w gives the total writhe of an oriented diagram of a two component link.

```
w1[PD[Xs___X]] :=
  (le = L[PD[Xs]];
   count[PD[Xs] /. {X[i_, j_, k_, l_] /; (j == l + 1 || l == le  && j == 1) && (i ≤ le && j ≤  le)
         :> 1,
       X[i_, j_, k_, l_] /; (l == j + 1 || j == le  && l == 1) && (i ≤ le && j ≤  le)
         :> - 1}])
```

w1 gives the total writhe of an oriented diagram of the first component of a two component link.

```
w2[PD[Xs___X]] :=
 (le = L[PD[Xs]];
   count[
   PD[Xs] /. {X[i_, j_, k_, l_] /; (j == l + 1 || (j == le + 1 && l ≠ le + 2 )) && (i > le && j > le)
      :→ 1,
     X[i_, j_, k_, l_] /; (l == j + 1 || (l == le + 1 && j ≠ le + 2 )) && (i > le && j > le)
       :→ -1}])
```

w2 gives the total writhe of an oriented diagram of the second component of a two component link.

```
Lk[PD[Xs___X]] :=
 ( w[PD[Xs]] - w1[PD[Xs]] - w2[PD[Xs]]) / 2
```

Lk gives the linking number

The following function returns a reduced a polynomial given a a laurent polynomial in A, that gives the same value assuming A^10=1.

```
p[x_] :=
  (xx = x; While [PolynomialQ[ xx // Simplify, A] // Not, xx = A^10 xx // Expand]; xx)  // r;
```

The following funcion allows one to obtain a canonical form for an element of the field,

```
r[a_] := PolynomialRemainder[a // Expand, Cyclotomic[10, A], A]
```

The following function allows one to divide by 1+A in the field.

```
d[a_] := (a - ((a /. A → -1) / 5) Cyclotomic[10, A]) / (A + 1) // Simplify // r
```

This gives the norm

```
n[a_] := Product[a /. A → (-1)^(i + 1) A^i, {i, 1, 4}] // r
```

This gives the inverse

```
Q[a_] := PolynomialRemainder[(a // p) /. A → -q^3 // Expand, Cyclotomic[5, q], q]
```

This changes to q= zeta_5

```
inv[a_] := (Product[a /. A → (-1)^(i + 1) A^i, {i, 2, 4}] // r) / n[a]
```

```
delta = -A^-2 - A^2 // p;
```

```
B1[f_, pd_] := p[(-A^3)^(f - w1[pd]) delta KB[pd] +
      KB[L2[pd]] + (-A^3)^(f - w1[pd]) 2 delta KB[L1[pd]] + 2] // d // d
```

```
B2[f_, pd_] := p[(-A^3)^(f - w2[pd]) delta KB[pd] +
      KB[L1[pd]] + (-A^3)^(f - w2[pd]) 2 delta KB[L2[pd]] + 2] // d // d
```

```
b1[f_, pd_] := p[ (-A^3)^(f - w1[pd])  delta KB[L1[pd]] + 1] // d
```

```
b2[f_, pd_] := p[ (-A^3)^(f - w2[pd])  delta KB[L2[pd]] + 1] // d
```

B1 and b1 are generators of the ideal of the 3-manifold obtained if surgery is done along L1 and a neighborhood of L2 is removed. B2 and b2 are generators of the ideal of the 3-manifold obtaind if surgery is done along L2 and a neighborhood of L1 is removed.The next functions g1 andg2 can detect that the ideal generated by B1and b1 (or B2 and b2) is trivial, or that an ideal included in (h). As the ring is a pid, the norm of the generator must divide the gcd of the norms of any collection of elements in the ideal. Also an element is a unit iff its norm is one. Thus if the output is one, then the ideal is trivial. If the output is 5, then the norm of the generator must divide 5. If the output is 5 and it is known that the ideal is contained in (h) with exponent greater than one, then the ideal is (h).

```
g1[f_, pd_] := GCD [(yy = B1[f, pd] ) // n, (zz = b1[f, pd]) // n, yy + zz // n, yy + 2 zz // n,
  2 yy + 3 zz // n,  yy + 4 zz // n,  yy - zz // n,  yy - 2 zz // n, 2 yy - 3 zz // n,  yy - 4 zz // n]
```

```
g2[f_, pd_] := GCD [(yy = B2[f, pd] ) // n, (zz = b2[f, pd]) // n, yy + zz // n, yy + 2 zz // n,
  2 yy + 3 zz // n,  yy + 4 zz // n,  yy - zz // n,  yy - 2 zz // n, 2 yy - 3 zz // n,  yy - 4 zz // n]
```

The input for t1 or t2 is a link name from Bar-Natan's Atlas. t1 checks for small ideals in the 3-manifold given by k framed surgery (as k varies through the residue classes mod 5: 0,1,2,3,4) to the first component and removing a

neighborhood of the second component. If the ideal is small it outputs :
the link type,
the framing k,
the linking number of the two components,
sometimes a polynomial in A
the norm of the generator of the small ideal,
followed by the generator of the small ideal written in terms of q.
If the polynomial in A is not integral then the ideal has not been calculated and this pair link and framing needs to be further investigated. (this happens only once below).
t2 does the same with the role of first and second component reversed.

```
t1[x_] := (
  If[
    (Mod[Lk[PD[x]], 5] == 0 && g1[0, PD[x]] ≠ 5) ||
     (Mod[Lk[PD[x]], 5] != 0 && g1[0, PD[x]] ≠ 1), If[n[zz] ≠ 0,
    Print[{x, 0, Lk[PD[x]],  yy * inv [zz] // p, zz // n, zz // Q}],
       Print[{x, 0, Lk[PD[x]],  yy // n, yy // Q }] ]];
  Do[
   If[g1[f, PD[x]] ≠ 1,
    If[ n[zz] ≠ 0, Print[{x, f, Lk[PD[x]], yy * inv [zz] // p, zz // n, zz // Q}],
      Print[{x, f, Lk[PD[x]],  yy // n, yy // Q }] ]]
   , {f, 1, 4}]
 )
t2[x_] := (
  If[
    (Mod[Lk[PD[x]], 5] == 0 && g2[0, PD[x]] ≠ 5) ||
     (Mod[Lk[PD[x]], 5] != 0 && g2[0, PD[x]] ≠ 1), If[n[zz] ≠ 0,
    Print[{x, 0, Lk[PD[x]],  yy * inv [zz] // p, zz // n, zz // Q}],
       Print[{x, 0, Lk[PD[x]],  yy // n, yy // Q }] ]];
  Do[
   If[g2[f, PD[x]] ≠ 1,
    If[ n[zz] ≠ 0, Print[{x, f, Lk[PD[x]], yy * inv [zz] // p, zz // n, zz // Q}],
      Print[{x, f, Lk[PD[x]],  yy // n, yy // Q }] ]]
   , {f, 1, 4}]
 )
```

Thus the links listed below (as output) are all the links within Thistlewaites list with small ideals that occur in Thistlewaites list. Each link is followed by a generator for this small ideal except Link[11,NonAlternating,89] with surgery along second component with framing 0 has ideal 1+2q^3, as noted at the end of the file.

```
Do[ t2[Link[7, Alternating, m]], {m, 1, 6}]

Do[ t1[Link[7, Alternating, m]], {m, 1, 6}]

Do[ t2[Link[7, NonAlternating, m]], {m, 1, 2}]

Do[ t1[Link[7, NonAlternating, m]], {m, 1, 2}]

Do[ t2[Link[8, Alternating, m]], {m, 1, 14}]

Do[ t1[Link[8, Alternating, m]], {m, 1, 14}]

Do[ t2[Link[8, NonAlternating, m]], {m, 1, 2}]

Do[ t1[Link[8, NonAlternating, m]], {m, 1, 2}]

Do[ t2[Link[9, Alternating, m]], {m, 1, 42}]
```

```
{Link[9, Alternating, 6], 0, -2, 31, -1 - 2 q^2 + q^3}
{Link[9, Alternating, 7], 1, -2, 2 - 2 A + 2 A^2, 11, 2 + q^3}
{Link[9, Alternating, 11], 1, -2, 1 + A^2 + A^3, 11, 1 + q - q^2 + 2 q^3}
{Link[9, Alternating, 12], 0, -2, 11, -2 - 2 q - q^2 - 2 q^3}
{Link[9, Alternating, 15], 4, 0, 1 + A^3, 11, 1 + q - q^2 + q^3}
{Link[9, Alternating, 17], 3, 0, -A + A^3, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[9, Alternating, 23], 3, -3, 3 - A + 2 A^2 - A^3, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[9, Alternating, 23], 4, -3, 3 - 2 A + 2 A^2 - 2 A^3, 11, 2 - q + q^2}
Do[ t1[Link[9, Alternating, m]], {m, 1, 42}]
Do[ t2[Link[9, NonAlternating, m]], {m, 1, 19}]
Do[ t1[Link[9, NonAlternating, m]], {m, 1, 19}]
Do[ t2[Link[10, Alternating, m]], {m, 1, 121}]
{Link[10, Alternating, 1], 1, 0, -1 + A + 3 A^3, 11, -1 + q - 2 q^2}
{Link[10, Alternating, 1], 2, 0, 1 + A + 2 A^2 + 2 A^3, 11, 3 + q^2 + 2 q^3}
{Link[10, Alternating, 10], 0, 0, 1 - A + A^2 + A^3, 25, 3 + q^2 + q^3}
{Link[10, Alternating, 12], 1, -2, 1 + A^2 + A^3, 11, -2 - q - 2 q^2 - 2 q^3}
{Link[10, Alternating, 12], 2, -2, 2 + 2 A^2 + A^3, 11, 2 + q + 3 q^3}
{Link[10, Alternating, 15], 2, -2, 2 - A + A^2 + A^3, 11, 1 - q^2 + q^3}
{Link[10, Alternating, 16], 3, -2, 1 - A + A^2 + A^3, 11, 3 + 2 q + q^2 + 3 q^3}
{Link[10, Alternating, 18], 0, 0, 25, 1 - q - q^2 + q^3}
{Link[10, Alternating, 24], 1, 0, 1 + A^2 + 2 A^3, 11, -1 + q - 2 q^2}
{Link[10, Alternating, 27], 3, 0, 1 + A^3, 11, 2 + q + q^3}
{Link[10, Alternating, 31], 1, 0, 2 - A + 2 A^2, 11, 2 - q + q^2 + q^3}
{Link[10, Alternating, 35], 0, 0, 55, -2 - 3 q^2}
{Link[10, Alternating, 41], 1, 0, -A + A^3, 11, -1 + q - 2 q^2}
{Link[10, Alternating, 43], 0, -2, 121, q - 3 q^3}
{Link[10, Alternating, 44], 0, -2, 11, -2 - 2 q - q^2 - 2 q^3}
{Link[10, Alternating, 49], 0, -2, 31, -2 - 3 q - q^2 - q^3}
{Link[10, Alternating, 68], 3, -1, 2 A^3, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[10, Alternating, 68], 4, -1, 1 + A^2 + A^3, 11, 2 - q + q^2}
{Link[10, Alternating, 76], 2, 1, 4 - 3 A + A^2 - 4 A^3, 11, 3 + q^2 + 2 q^3}
Do[ t1[Link[10, Alternating, m]], {m, 1, 121}]
Do[ t2[Link[10, NonAlternating, m]], {m, 1, 64}]
```

```
{Link[10, NonAlternating, 16], 0, 0, 55, -1 - q^2 + 2 q^3}
{Link[10, NonAlternating, 25], 3, -2, 2 - 2 A + 2 A^2 - A^3, 11, 2 + q + q^3}
{Link[10, NonAlternating, 28], 1, -2, 2 - 2 A + A^2 - A^3, 11, 2 - q + q^2 + q^3}
{Link[10, NonAlternating, 32], 0, 0, 2 - A, 25, q - 2 q^2 + q^3}
{Link[10, NonAlternating, 40], 1, 1, 2 - A + A^2, 11, -1 + q - 2 q^2}
{Link[10, NonAlternating, 52], 4, -1, 3 - A + A^2 - A^3, 11, 3 + q + q^2 + 2 q^3}
{Link[10, NonAlternating, 62], 4, 0, 1 - A + A^2, 11, 2 - q + q^2}
```

Do[t1[Link[10, NonAlternating, m]], {m, 1, 64}]

```
{Link[10, NonAlternating, 62], 4, 0, 1 - A + A^2, 11, 2 - q + q^2}
```

Do[t2[Link[11, Alternating, m]], {m, 1, 384}]

```
{Link[11, Alternating, 14], 0, 0, 55, 1 + 2 q - q^2 - 2 q^3}
{Link[11, Alternating, 17], 0, 0, 155, 2 + 3 q - q^2 + q^3}
{Link[11, Alternating, 46], 0, 0, 1 - 3 A - A^2 - A^3, 25, q - 2 q^2 + q^3}
{Link[11, Alternating, 46], 1, 0, -1 - 3 A - 2 A^2, 11, 2 - q + q^2 + q^3}
{Link[11, Alternating, 61], 2, -2, A + A^3, 11, -q - q^2 - 2 q^3}
{Link[11, Alternating, 71], 3, -2, 1 + A - A^2 + 2 A^3, 11, -1 - q - 2 q^2 - 2 q^3}
{Link[11, Alternating, 75], 3, -2, 1 + A^2, 11, -1 - 2 q - q^2 - 2 q^3}
{Link[11, Alternating, 84], 4, -2, -A + 2 A^3, 11, 1 + q - q^2 + q^3}
{Link[11, Alternating, 85], 0, -2, 11, -1 - 2 q - 2 q^2 - 2 q^3}
{Link[11, Alternating, 89], 2, -2, 2 - A + A^2 + A^3, 11, -2 q - q^2 - q^3}
{Link[11, Alternating, 97], 0, -2, 61, -3 - 3 q - 2 q^2 + q^3}
{Link[11, Alternating, 109], 4, -2, 2 - A + 2 A^2 + A^3, 11, 1 - q + q^2 + q^3}
{Link[11, Alternating, 111], 2, 0, 1 + A^3, 11, -1 - q^2 - 2 q^3}
{Link[11, Alternating, 117], 1, 0, 2 - A + 2 A^2, 11, 2 - q + q^2 + q^3}
{Link[11, Alternating, 120], 3, -2, 1 + 2 A^2 + A^3, 11, 3 + q + 2 q^2 + 3 q^3}
{Link[11, Alternating, 123], 0, -2, 61, 2 - q^2 + 2 q^3}
{Link[11, Alternating, 125], 0, 0, 55, -2 - 3 q^2}
{Link[11, Alternating, 128], 3, -2, -3 - A^2 + 4 A^3, 11, 4 + 3 q^2 + 2 q^3}
{Link[11, Alternating, 131], 4, 0, -A + A^3, 11, 2 - q + q^2}
{Link[11, Alternating, 133], 2, -2, 2 + 2 A^2 + A^3, 11, 1 - q^2 + q^3}
{Link[11, Alternating, 136], 4, -2, A^2 + 2 A^3, 11, 3 + q + q^2 + 2 q^3}
{Link[11, Alternating, 146], 3, -1, 4 - A + 2 A^2 - A^3, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[11, Alternating, 153], 0, -3, 11, 1 + 2 q + 3 q^2 + 2 q^3}
{Link[11, Alternating, 158], 1, -1, -1 - 2 A - A^2 + A^3, 11, 1 + q - q^2 + 2 q^3}
```

```
{Link[11, Alternating, 164], 0, -3, 11, -2 - 2 q - 2 q^2 - q^3}
{Link[11, Alternating, 186], 0, -3, 41, -1 + 2 q + q^2 + q^3}
{Link[11, Alternating, 188], 0, -1, 31, 1 + 2 q + q^2 + 3 q^3}
{Link[11, Alternating, 198], 1, 1, -1 - 2 A - 2 A^2, 11, -1 + q - 2 q^2}
{Link[11, Alternating, 200], 0, -3, 31, -1 - 2 q^2 + q^3}
{Link[11, Alternating, 208], 0, 1, 211, 2 + 3 q + 2 q^2 + 5 q^3}
{Link[11, Alternating, 213], 4, -1, -2 A - A^2, 11, 1 + q - q^2 + q^3}
{Link[11, Alternating, 214], 4, -3, 4 - A + 2 A^2 - 2 A^3, 11, 1 + q - q^2 + q^3}
{Link[11, Alternating, 224], 0, -1, 61, 3 q + 2 q^2 + 2 q^3}
{Link[11, Alternating, 227], 1, -3, 6 - 3 A + 3 A^2 - 4 A^3, 11, 1 + q - q^2 + 2 q^3}
{Link[11, Alternating, 228], 4, -1, 5 - A + 3 A^2 - 2 A^3, 11, 2 - q + q^2}
{Link[11, Alternating, 229], 0, -3, 71, -3 - 4 q - 3 q^2 - 2 q^3}
{Link[11, Alternating, 235], 0, -3, 61, 2 + 2 q - q^2}
{Link[11, Alternating, 244], 4, -3, 6 - A + 4 A^2 - 3 A^3, 11, 2 - q + q^2}
{Link[11, Alternating, 273], 1, -4, 1 - 2 A - A^3, 11, -1 + q - 2 q^2}
{Link[11, Alternating, 273], 2, -4, -2 A, 11, 3 + q^2 + 2 q^3}
{Link[11, Alternating, 282], 2, 2, 2 + 2 A^2 + A^3, 11, 3 + q^2 + 2 q^3}
{Link[11, Alternating, 304], 3, -4, 2 + A^2, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[11, Alternating, 308], 1, -2, 1 - 3 A - 2 A^3, 11, -1 + q - 2 q^2}
{Link[11, Alternating, 317], 2, 0, 2 + 2 A + 3 A^2 + 3 A^3, 11, 3 + q^2 + 2 q^3}
{Link[11, Alternating, 318], 2, 0, 2 - A + A^2 - A^3, 11, 3 + q^2 + 2 q^3}
{Link[11, Alternating, 327], 3, -2, 2 - 3 A - 2 A^3, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[11, Alternating, 329], 4, -2, -3 A - A^3, 11, 2 - q + q^2}
{Link[11, Alternating, 330], 4, 0, 1 + A + 2 A^2 + 2 A^3, 11, 2 - q + q^2}
{Link[11, Alternating, 339], 3, 0, -1 + A + 3 A^3, 11, 1 + 2 q - q^2 + 2 q^3}
{Link[11, Alternating, 342], 4, -2, 2 + A + 2 A^2 + 2 A^3, 11, 3 + q + q^2 + 2 q^3}
{Link[11, Alternating, 343], 2, 0, 4 + 3 A^2, 11, 3 + q^2 + 2 q^3}
```

Do[t1[Link[11, Alternating, m]], {m, 1, 384}]

```
{Link[11, Alternating, 342], 1, -2, -1 + A + A^2 + 3 A^3, 11, -1 + q - 2 q^2}
{Link[11, Alternating, 343], 2, 0, 5 - 2 A + 2 A^2 - 4 A^3, 11, 3 + q^2 + 2 q^3}
{Link[11, Alternating, 344], 1, -2, -1 + A + 3 A^3, 11, -1 + q - 2 q^2}
```

Do[t2[Link[11, NonAlternating, m]], {m, 1, 254}]

```
{Link[11, NonAlternating, 4], 2, -2, 1 - A - A^3, 11, -2 - q - q^3}
{Link[11, NonAlternating, 11], 2, 0, 1 - A + 2 A^2 - A^3, 11, -q - q^2 - 2 q^3}
```

$\{$Link[11, NonAlternating, 20], 1, 0, $1 + A^3$, 11, $-1 + q - q^2 - q^3\}$

$\{$Link[11, NonAlternating, 33], 3, 0, $2 - A + A^2 - A^3$, 11, $-1 + q^2 - q^3\}$

$\{$Link[11, NonAlternating, 37], 1, 0, $2 - 2A - A^3$, 11, $q + 2q^3\}$

$\{$Link[11, NonAlternating, 39], 3, 0, $2 - A + A^2 - A^3$, 11, $-1 - q^2 + q^3\}$

$\{$Link[11, NonAlternating, 40], 1, −2, $2 - 2A + A^2 - A^3$, 31, $3 + 2q + q^2 + 2q^3\}$

$\{$Link[11, NonAlternating, 42], 2, −2, $1 - 2A$, 11, $2 + q + 3q^3\}$

$\{$Link[11, NonAlternating, 47], 1, −2, $2 - 2A + A^2 - A^3$, 11, $-1 + q - q^2 - q^3\}$

$\{$Link[11, NonAlternating, 52], 0, 0, $2 - A + A^2$, 25, $q - 2q^2 + q^3\}$

$\{$Link[11, NonAlternating, 54], 3, −2, $2 - 2A + 2A^2 - A^3$, 11, $-1 - q - 2q^2 - 2q^3\}$

$\{$Link[11, NonAlternating, 58], 0, −2, 31, $-2q - q^2 + q^3\}$

$\{$Link[11, NonAlternating, 59], 0, −2, 16, $2 + 2q + 2q^2 + 2q^3\}$

$\{$Link[11, NonAlternating, 65], 1, −2, $2 - 2A + A^2 - A^3$, 11, $-2 - q - 2q^2 - 2q^3\}$

$\{$Link[11, NonAlternating, 68], 3, −2, $2 - 2A + 2A^2 - A^3$, 11, $-1 - 2q - q^2 - 2q^3\}$

$\{$Link[11, NonAlternating, 70], 1, −2, $2 - 2A + A^2 - A^3$, 11, $-1 + q - 2q^2\}$

$\{$Link[11, NonAlternating, 74], 1, −2, $2 - 2A + A^2 - A^3$, 11, $-1 + q - 2q^2\}$

$\{$Link[11, NonAlternating, 76], 1, −2, $2 - 3A - A^3$, 11, $q + 2q^3\}$

$\{$Link[11, NonAlternating, 79], 2, −2, $3 - 3A + A^2 - A^3$, 11, $2 + q + 2q^2 + q^3\}$

$\{$Link[11, NonAlternating, 82], 4, −2, $3 - 3A + A^2 - 2A^3$, 11, $2q - q^2 + q^3\}$

$\{$Link[11, NonAlternating, 86], 0, −2, 11, $1 + 2q^3\}$

$\{$Link[11, NonAlternating, 89], 0, −2, $\frac{9}{5} - \frac{8A}{5} + \frac{7A^2}{5} - \frac{6A^3}{5}$, 55, $-1 - 3q^2 - q^3\}$

$\{$Link[11, NonAlternating, 89], 2, −2, $2 - 2A + 2A^2 - 2A^3$, 11, $-2 - q - q^3\}$

$\{$Link[11, NonAlternating, 90], 1, −2, $2 - A + 2A^2 - A^3$, 11, $2 - q + q^2 + q^3\}$

$\{$Link[11, NonAlternating, 91], 2, 0, $2 - 2A - A^3$, 11, $2 + q + 3q^3\}$

$\{$Link[11, NonAlternating, 94], 3, 0, $2 - A^2 + A^3$, 11, $q - q^2 - q^3\}$

$\{$Link[11, NonAlternating, 103], 1, −2, $2 - 2A + A^2 - A^3$, 31, $1 - 2q - q^3\}$

$\{$Link[11, NonAlternating, 112], 0, 0, $2 - 2A + A^2 - A^3$, 55, $3 - q + q^2 + 2q^3\}$

$\{$Link[11, NonAlternating, 115], 4, −2, $2 - A + A^2$, 71, $-1 + q - 3q^2\}$

$\{$Link[11, NonAlternating, 121], 0, −2, 11, $2q + q^3\}$

$\{$Link[11, NonAlternating, 123], 0, −2, 11, $-2 - 2q - 2q^2 - q^3\}$

$\{$Link[11, NonAlternating, 124], 0, 0, $2 - 2A + A^2 - A^3$, 55, $2 - q - q^3\}$

$\{$Link[11, NonAlternating, 125], 4, −2, $2 - A + A^2$, 61, $2 + 3q + 2q^3\}$

$\{$Link[11, NonAlternating, 131], 0, −1, 11, $-1 - 2q^2\}$

$\{$Link[11, NonAlternating, 137], 1, −1, $4 - 2A + A^2 - 2A^3$, 11, $-1 + q - 2q^2\}$

{Link[11, NonAlternating, 154], 0, -1, 31, $-1+2q+q^3$}

{Link[11, NonAlternating, 155], 0, -1, 11, $1+2q+2q^2+2q^3$}

{Link[11, NonAlternating, 162], 4, -3, $2-A+A^2$, 11, $1+q-q^2+q^3$}

{Link[11, NonAlternating, 169], 0, -3, 11, $2q+q^3$}

{Link[11, NonAlternating, 170], 0, -1, 121, $-3-4q^2-q^3$}

{Link[11, NonAlternating, 171], 4, -1, $4-3A+2A^2-2A^3$, 11, $1+q-q^2+q^3$}

{Link[11, NonAlternating, 174], 4, -3, $1-2A+A^2$, 11, $3+q+q^2+2q^3$}

{Link[11, NonAlternating, 176], 2, -3, $1-A+A^2$, 11, $3+q^2+2q^3$}

{Link[11, NonAlternating, 181], 4, -3, $2-A+A^2$, 11, $2-q+q^2$}

{Link[11, NonAlternating, 190], 2, -3, $-1-A-A^2+A^3$, 11, $3+q^2+2q^3$}

{Link[11, NonAlternating, 191], 4, -1, -A, 11, $2-q+q^2$}

{Link[11, NonAlternating, 212], 4, -4, -A, 11, $2-q+q^2$}

{Link[11, NonAlternating, 215], 1, -2, $1+A^2+A^3$, 11, $-1+q-2q^2$}

{Link[11, NonAlternating, 225], 1, 0, $-A-A^2$, 11, $-1+q-2q^2$}

Do[t1[Link[11, NonAlternating, m]], {m, 1, 254}]

{Link[11, NonAlternating, 240], 3, 0, $2-A+A^2-A^3$, 11, $1+2q-q^2+2q^3$}

B2[0, Link[11, NonAlternating, 89] // PD]

$A-4A^3$

so B2= A (1-2A)(1+2A)

b2[0, Link[11, NonAlternating, 89] // PD]
$2-2A+3A^2-3A^3$

$2-2A+3A^2-3A^3$

$2-2A+3A^2-3A^3$ // d

A^2-2A^3

so b2=A^2 (1-2A)(1+A)

The ideal generated by b2 and B2 is (1-2A) times the ideal generated by 1+A and 1+2A.
so FKB ideal of Link[11,NonAlternating,89] with K the first component with framing 0 is generated by 1-2A=1+2q^3