Knot invariant and the Reidemeister theorem

LSU Math: Informal Geometry & Topology Seminar

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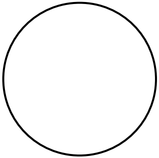
Outline

Motivation

This talk draws motivation from "On Ribbon graphs and Virtual links" a paper by S. Baldridge, L. Kauffman, and W. Rushworth. (2022)

Knots and Links

Consider the diagram



a. unknot

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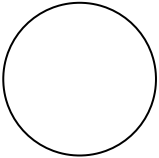

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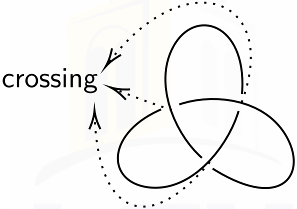

b. trefoil

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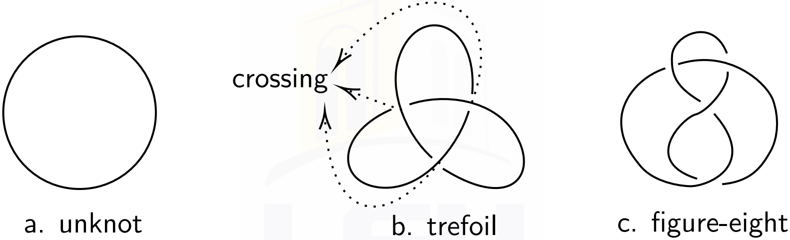

Knots

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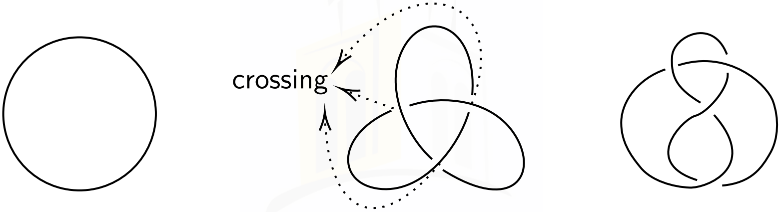

a. unknot   b. trefoil   c. figure-eight

Examples of **Knots**.

Knots

Definition 1 (knot)

A *knot* is a closed, non-self-intersecting curve (S^1) smoothly embedded in three-dimensional space (i.e. $\mathbb{R}^3$).

Knots and Links

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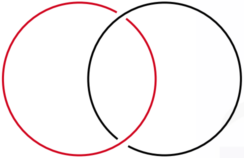

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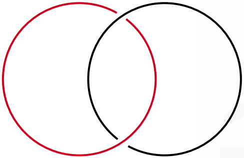

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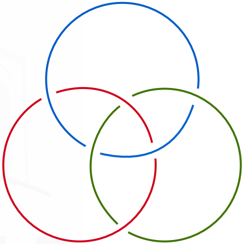

b. Borromean rings link

Links

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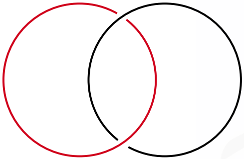

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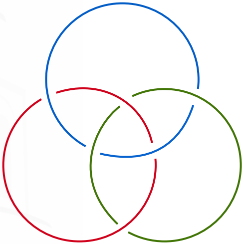

b. Borromean rings link

Examples of **Links**.

Links

Definition 2 (Link)

A *link* is a collection of one or more knots that may be entangled.

Knot Invariant

Definition 3 (Knot Invariant)

A *knot invariant* is a property or quantity associated with a knot that remains unchanged under ambient isotopy (i.e. continuous deformation without cutting or passing through itself).

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- Reidemeister moves.

Other examples; Alexander polynomial, Jones polynomial, Khovanov homology, and many more.

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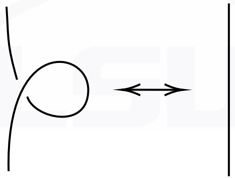

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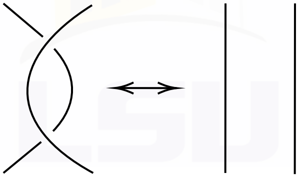

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The Reidemeister Theorem

Theorem 8 (Reidemeister Theorem)

Two knot diagrams are topologically equivalent if and only if one can be transformed into the other by a finite sequence of Reidemeister moves (Move I, II, and III).

Pictorial prove of the Reidemeister Theorem

***Proof*.** Consider the diagrammatic proof below.

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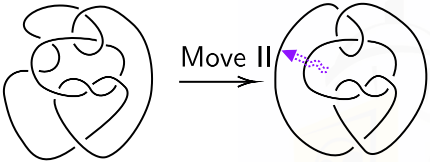

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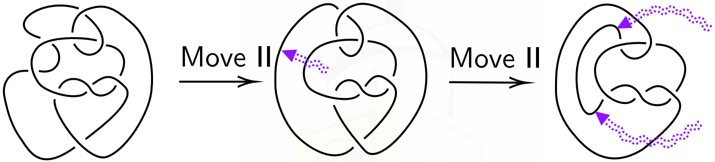

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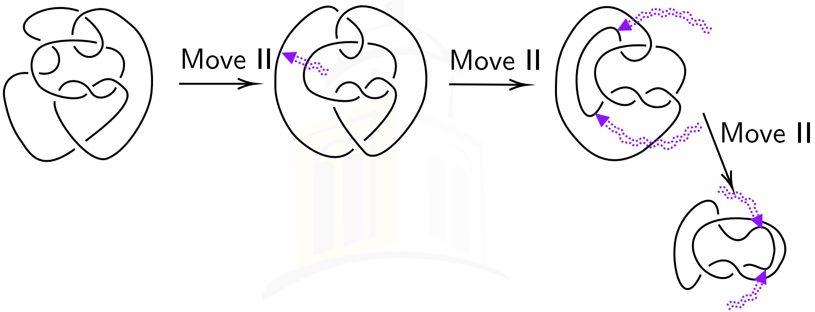

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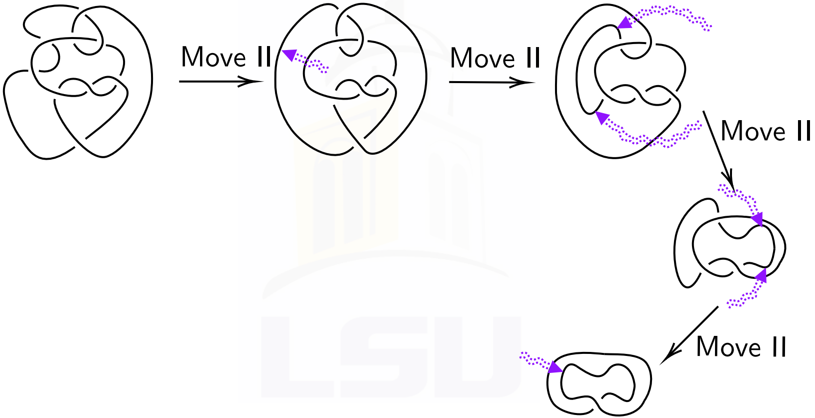

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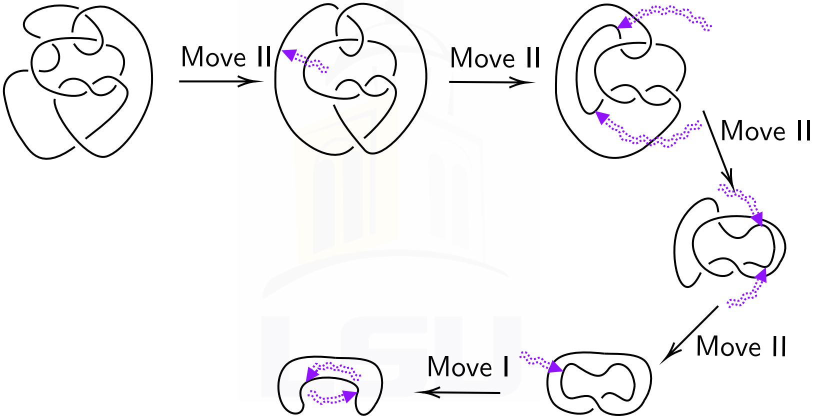

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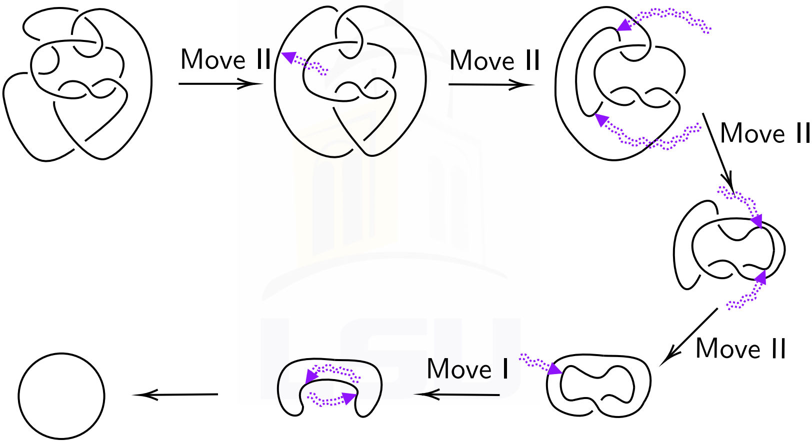

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References

[CAdms] C. Adams. *The Knot Book.* W.H. Freeman and Company, 1994.

[DRolf] D. Rolfsen. *Knots and Links.* AMS Chelsea Publishing UK ed. Edition, 2023.

End of Presentation

Thank you.