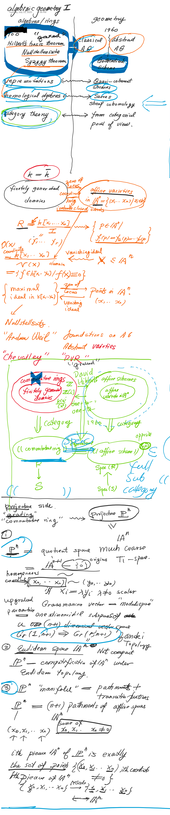

Noetherian ring
"Arithmetic Weil" $\rightsquigarrow$ Projective $\mathbb{P}^n$
$\mathbb{P}^n$
quotient space with Coarse π_1-space
$\mathbb{P}^n = \frac{\mathbb{A}^n - \{(0,\dots,0)\}}{\mathbb{G}_m}$
homogeneous coordinates $(x_0 : \dots : x_n)$
$\sim (\lambda x_0 : \dots : \lambda x_n)$
$\forall \lambda \in \mathbb{G}_m$ λ is the scalar
upgraded Grassmannian vector "moduli space"
parameter space of all subspaces of dim k in $\mathbb{A}^n$
$\mathbb{G}r(k,n) = \mathrm{Gr}(k,n)$
$\mathbb{G}r(k,n) \Rightarrow \mathrm{Gr}(k,n)$ topology
① Quotient space $\mathbb{G}r(1,\mathbb{A}^n)$ $\mathbb{P}^n$ compact
$\mathbb{P}^n$ — compactification of $\mathbb{A}^n$ under Euclidean topology
② $\mathbb{P}^n$ "manifold" — patchwork of transition functions
$\mathbb{P}^n = (n+1)$ patchwork of affine spaces
$(x_0, x_1, \dots, x_n)$
$\frac{\text{part of } x_0}{x_0}, \frac{\text{part of } x_1}{x_1}, \dots, \frac{\text{part of } x_n}{x_n}$
the point $\mathbb{A}^n$ of $\mathbb{P}^n$ is exactly the set of points $\{(x_0, x_1, \dots, x_n)\}$ satisfied the following $\mathbb{A}^n$ point $\{x_0, x_1, \dots, x_n\} \mapsto \{\frac{x_1}{x_0}, \dots, \frac{x_n}{x_0}\}$
$\longleftrightarrow \mathbb{A}^n$

"Weil conjecture"
Weil cohomology
étale cohomology

Def: ((schemes))

- Objective schemes = "locally ringed spaces"
  locally isomorphic to some affine schemes Spec(R)
- morphism morphisms of "locally ringed spaces"

((commutative rings))^{op} $\xrightarrow{\text{Spec}}$ ((affine schemes))

$\cap$

((schemes))

moduli space

$\cup$

((algebraic spaces)) $\cap$ ((locally ringed space))

$\cup$ $\cap$

((stacks)) ((topos))

X topology $\longrightarrow$ $Sh(X)$ — abelian category

R ring $\longrightarrow$ $mod(R)$ $\rightarrow$ abelian category

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topological way of defining a sheaf of sets of abelians

E étal space $p: E \to X$ locally homeomorphic.

$p \downarrow$

X

Karen Smith

(pre)

A sheaf on X is assignment

open set in X, U $F(U)$ abelian groups

$V \xrightarrow[\text{inclusion}]{\text{open}} U$ functorial

$F(V) \xleftarrow[\text{restriction}]{\rho_{VU}} F(U)$ $\begin{cases} \rho_{UU} = \mathrm{Id} \\ \rho_{WV}\rho_{VU} = \rho_{WU} \end{cases}$

$\cap$

$s|_V \longleftarrow s$ sections on U

$\shortparallel$

$\rho_{VU}(s)$

A sheaf $\mathcal{F}$ on X is a presheaf which "sheaf properties" = data is locally restorable

Given sections on any open cover U_i of an open set U, satisfying the compatible condition

$s_i \in U_i$, $s_j \in U_j$

$s_i|_{U_i \cap U_j} = s_j|_{U_j \cap U_j}$

glueing there is a $\boxed{\text{UNique}}$ section s in U s.t. $\boxed{s|_{U_i} = s_i}$

$\left\{ \begin{array}{c} E \\ \downarrow \\ X \end{array} \right\} \longleftrightarrow \left\{ \boxed{\mathcal{F}op(X)^{op} \to Ab} \right\}$ functors

$\boxed{\text{satisfies sheaf property}}$

"Not Hausdorff" ob open sets

morphism open inclusion

$E \xrightarrow{p} X$

$X \supset U$

sheaf of sections on U

$F(U) = \left\{ \begin{array}{c} \text{section} \\ U \xrightarrow{i} E \\ p_i = \mathrm{Id}_U \end{array} \right\}$

$E = \bigcup_{x \in X} F_x$ continuous

$\downarrow$

X

$\longleftarrow$ sheaf on X

$F_x = \varinjlim_{U \ni x} F(U)$ $f \sim g$ agree on smaller open set