Math 1550-24 Review and old test for Exam (1)

Review the following items:

1. Slope of a tangent line to a curve
2. Limit of a function
3. One-sided limit
4. Infinite limit
5. Vertical asymptote
6. Limit laws
7. Squeeze theorem
8. Continuity
9. Right-continuity, left-continuity
10. Continuous functions
11. Polynomials, rational functions, algebraic functions, trigonometric function, inverse trigonometric functions, exponential functions, logarithmic functions
12. Absolute value function, greatest integer function
13. Limit at infinity
14. Horizontal asymptote
15. Tangent, velocity, rates of change
16. Derivative

Practive the following problems from an old test:

1. Evaluate the function $f(x) = \dfrac{1-\cos x}{x^2}$ at the numbers $x = 0.1, 0.01, 0.001, 0.0001$ to guess the value of $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x^2}$.
2. Evaluate the limit $\displaystyle\lim_{x\to 2} \frac{x^2-3x+2}{x^2-x-2}$.
3. Evaluate the limit $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{x\sqrt{1+x}}\right)$.
4. Evaluate the limit $\displaystyle\lim_{x\to 7^-} \Big([\![x]\!] + [\![3-x]\!]\Big)$.
5. Suppose a function $f(x)$ is defined by $f(x) = \begin{cases} x+2, & \text{if } x < 3, \\ a, & \text{if } x = 3, \\ x^2+bx, & \text{if } x > 3. \end{cases}$
   Find the values of a and b so that $f(x)$ is continuous on $(-\infty, \infty)$.
6. Find the vertical asymptote(s) of the curve $y = \dfrac{x^3-1}{x^4-4x^2}$.
7. Evaluate the limit $\displaystyle\lim_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2+1}\right)$.
8. Find the horizontal asymptote(s) of the curve $\dfrac{1-|x|}{1+2x}$.
9. Evaluate the limit $\displaystyle\lim_{x\to\infty} \frac{1-\sqrt{x}}{1+\sqrt{x}}$.
10. Use the **definition** of derivative to find $f'(a)$ for the function $f(x) = 3+\sqrt{x}$. Here a is any fixed real number.

Math 1550-24 Review and old test for Exam (2)

Review the following items:

1. Review Trigonometry in Appendix D.
2. Handout note: Memorize the derivative formulas and trigonometric identities.
3. Differentiability of a function at a point.
4. Differentiable functions.
5. Constant rule, sum rule, difference rule, constant multiple rule.
6. Power rule, product rule, quotient rule.
7. Chain rule.
8. Rates of change.
9. Implicit differentiation.
10. Important limits: $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1,\ \lim_{h\to 0}\frac{e^h-1}{h}=1,\ \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e.$
11. Several classes of functions:
    (1) Polynomials
    (2) Rational functions
    (3) Algebraic functions
    (4) Trigonometric functions
    (5) Exponential functions
    (6) Inverse trigonometric functions $\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$.)
12. Higher derivatives.

Practive the following problems from an old test:

1. Find the x-coordinates of the points on the curve $f(x)=x^3-3x^2+25$ where the tangent line has a slope 6.
2. Let $f(x)=\frac{x}{1+x^2}$. Find $f'(2)$.
3. Let $f(x)=(x^3+2x+3)e^{2x}$. Find $f'(0)$.
4. The position function of a particle is given by $s=t^3+t^2-5t,\ t\ge 0$. When does the particle reach a velocity of $3\,\mathrm{m/sec}$?
5. Find an equation of the tangent line to the curve $y=\sqrt[3]{2x^2+4}$ at the point $(\sqrt{2},2)$.
6. (8 pts) Evaluate the limit $\lim_{\theta\to 0}\frac{\sin\theta}{2\theta+\tan\theta}$.
7. Find all points on the graph of the function $f(x)=\sin^2 x+3\cos x$ at which the tangent line is horizontal.
8. Use the implicit differentiation to find $\frac{dy}{dx}$ for the curve $2^x+y^2=xy$.
9. Let $f(x)=\sin^{-1}(2x-1)+\tan^{-1}(x^2)$. Find $f'(\frac{1}{2})$.
10. Let $f(x)=\sec x$. Find $f''(\frac{\pi}{6})$.

Answers: 1. $1\pm\sqrt{3}$; 2. $-\frac{3}{25}$; 3. 8; 4. $\frac{4}{3}$ sec; 5. $y=\frac{\sqrt{2}}{3}x+\frac{4}{3}$;
6. $\frac{1}{3}$; 7. $x=n\pi$, n : integer; 8. $\frac{y-2^x\ln 2}{2y-x}$; 9. $\frac{50}{17}$; 10. $\frac{10}{9}\sqrt{3}$.

Math 1550-24 Review and old test for Exam (3)

Review the following items:

1. Memorize the derivative formulas.
2. Logarithmic differentiation.
3. Definitions of hyperbolic functions.
4. Graphs of $\sinh x, \cosh x, \tanh x$.
5. Derivatives of hyperbolic functions.
6. Do all homework problems on related rates.
7. Linear approximation $f(x) \approx f(a) + f'(a)(x-a)$ for x near a.
8. Increments and differentials.
9. Absolute maximum, absolute minimum, local maximum, local minimum.
10. Extreme value theorem.
11. Critical numbers and Fermat's thorem.
12. The closed interval method.
13. Rolle's theorem and the mean value theorem.
14. Increasing/decreasing test.
15. The first derivative test.
16. Concavity test.
17. The second derivative test.
18. Indeterminate forms and the L'Hospital's rule.

Practive the following problems from an old test:

1. Let $f(x) = \tan^{-1}(2x) + \sin^{-1}(\sqrt{x})$. Find $f'(x)$, but do not simplify.
2. Evaluate the limit $\displaystyle\lim_{n\to\infty}\left(\frac{n+2}{n}\right)^n$.
3. Solve the equation $\cosh x - \sinh x = 3$.
4. Let $y = x^{2x}$. Find $\left.\dfrac{dy}{dx}\right|_{x=e}$.
5. Two cars start moving from the same point. One travels north at 30 mi/h and the other travels east at 40 mi/h. At what rate is the distance between the cars increasing 30 minutes later?
6. Let $y = \log_2|7-3x|$. Find dy.
7. Find the critical numbers of the function $f(x) = x^{1/3}(x+1)$.
8. Find the absolute maximum and absolute minimum values of the function $f(x) = x + \dfrac{4}{x^2}$, $1 \le x \le 4$.
9. Find the interval(s) on which the function $f(x) = x^2 e^{-x}$ is decreasing.
10. Use the second derivative test to find the local maximum and minimum values of the function $f(x) = x - 2\sin x$, $0 \le x \le \pi$.
11. State the Extreme Value Theorem.

Answers: 1. $\frac{1}{1+(2x)^2}\cdot 2 + \frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{1}{2\sqrt{x}}$; 2. e^2; 3. $-\ln 3$; 4. $4e^{2e}$; 5. 50 mi/hr; 6. $\frac{-3}{(7-3x)\ln 2}\,dx$; 7. 0 and $-1/4$; 8. max $= 5$, min $= 3$; 9. $(-\infty, 0)$ and $(2,\infty)$; 10. Local min $\frac{\pi}{3} - \sqrt{3}$ at $x = \frac{\pi}{3}$, no local max.

Math 1550-24 Review and old test for Exam (4)

Review the following items:

1. Memorize the derivative and indefinite integral formulas.
2. Do all homework problems.
3. Review class notes.
4. Curve sketching.
5. Optimization problems.
6. Anti-derivatives.
7. Definition of area.
8. Sums of $\sum_{i=1}^{n} i$, $\sum_{i=1}^{n} i^2$, $\sum_{i=1}^{n} i^3$.
9. The definite integrals: partition, evaluation points, Riemann sum, limit.
10. The Fundamental Theorem of Calculus: Part 1 and Part 2.
11. Indefinite integrals.
12. Subtitution rule.

Practive the following problems from an old test:

1. Find point(s) of inflection on the curve $f(x) = xe^{-x}$.
2. A piece of wire 1 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a minimum? You must check that it is really a minimum.
3. Find $f(x)$ satisfying $f'(x) = 3^x$, $f(0) = 0$.
4. Let $f(x) = 3x$, $0 \le x \le 2$. Consider the partition $\{0 < \frac{2}{3} < 1 < 2\}$ and the evaluation set $\{\frac{1}{3}, 1, 2\}$. Find the resulting Riemann sum.
5. Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3(x_i^*)^2 - 5x_i^*\right)\Delta x$ as a definite integral for $0 \le x \le 2$.
6. Let $f(x) = \int_{-1}^{2x+1} \sqrt{1+t^2}\, dt$. Find $f'(1)$.
7. Evaluate the integral $\int_0^1 x\left(\sqrt[3]{x} - \sqrt{x}\right) dx$.
8. Evaluate the integral $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$.
9. Evaluate the integral $\int_0^{\ln 2} e^{-x}\, dx$.
10. Evaluate the integral $\int_0^{\pi/4} \sec x \tan x\, dx$.
11. Evaluate the integral $\int \cot^2 x\, dx$.
12. Evaluate the integral $\int \left(\frac{1}{\sqrt{2-x^2}} + \frac{1}{5+x^2}\right) dx$.

Answers: 1. $(2, 2e^{-2})$; 2. $\frac{4}{4+\pi}$ m long is bent into a square; 3. $\frac{1}{\ln 3}(3^x - 1)$; 4. 23/3;
5. $\int_0^2 (3x^2 - 5x)\, dx$; 6. $2\sqrt{10}$; 7. 1/35; 8. $\frac{1}{2}x^2 - 2x + \ln x + C$;
9. 1/2; 10. $\sqrt{2} - 1$; 11. $-\cot x - x + C$; 12. $\sin^{-1}\frac{x}{\sqrt{2}} + \frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + C$.