Math 1550-24 Review and old test for Exam (1)

Review the following items:

1. Slope of a tangent line to a curve
2. Limit, right-hand limit, and left-hand limit
3. Infinite limit and vertical asymptote
4. Limit laws and squeeze theorem
5. Continuity, right-continuity, and left-continuity
6. Continuous functions and intermediate value theorem
7. Polynomials, rational functions, algebraic functions, trigonometric function, inverse trigonometric functions, exponential functions, logarithmic functions
8. Absolute value function, greatest integer function
9. Limit at infinity and horizontal asymptote
10. Tangent, velocity, rates of change
11. Derivative and differentiability of a function at a point
12. Differentiable functions
13. Constant rule, sum rule, difference rule, constant multiple rule, power rule
14. Derivatives of polynomials and exponential functions.
15. $a^2 - b^2 = (a+b)(a-b)$
16. $(a+b)^2 = a^2 + 2ab + b^2$
17. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
18. $\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$
19. $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

Practice the following problems from an old test:

1. Evaluate the function $f(x) = \frac{1-\cos x}{x^2}$ at the numbers $x = 0.1, 0.01, 0.001, 0.0001$ to guess the value of $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.
2. Evaluate the limit $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-x-2}$.
3. Evaluate the limit $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{x\sqrt{1+x}}\right)$.
4. Evaluate the limit $\lim_{x\to 7^-} \left([\![x]\!] + [\![3-x]\!]\right)$.
5. Suppose a function $f(x)$ is defined by $f(x) = \begin{cases} x+2, & \text{if } x < 3, \\ a, & \text{if } x = 3, \\ x^2 + bx, & \text{if } x > 3. \end{cases}$
   Find the values of $a$ and $b$ so that $f(x)$ is continuous on $(-\infty, \infty)$.
6. Find the vertical asymptote(s) of the curve $y = \frac{x^3-1}{x^4-4x^2}$.
7. Evaluate the limit $\lim_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2+1}\right)$.
8. Find the horizontal asymptote(s) of the curve $\frac{1-|x|}{1+2x}$.
9. Evaluate the limit $\lim_{x\to\infty} \frac{1-\sqrt{x}}{1+\sqrt{x}}$.
10. Use the **definition** of derivative to find $f'(a)$ for the function $f(x) = 3 + \sqrt{x}$. Here $a$ is any fixed real number.
11. Find the $x$-coordinates of the points on the curve $f(x) = x^3 - 3x^2 + 25$ where the tangent line has a slope 6.
12. The position function of a particle is given by $s = t^3 + t^2 - 5t,\ t \geq 0$. When does the particle reach a velocity of 3 m/sec?

Math 1550-24 Review and old test for Exam (2)

Review the following items:

1. Trigonometry in Appendix D.
2. Handout notes: Memorize the derivative formulas and trigonometric identities.
3. Power rule, product rule, quotient rule.
4. Rates of change.
5. Important limits: $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1,\ \lim_{h\to 0}\frac{e^h-1}{h}=1,\ \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e.$
6. Derivatives of trigonometric functions.
7. Chain rule.
8. Implicit differentiation.
9. Derivatives of inverse trigonometric functions.
10. Higher derivatives.
11. Derivative of logarithmic functions.
12. Logarithmic differentiation.
13. Definitions of hyperbolic functions.
14. Graphs of $\sinh x, \cosh x, \tanh x$.
15. Derivatives of hyperbolic functions.
16. Word problems on related rates.
17. Linear approximation $f(x)\approx f(a)+f'(a)(x-a)$ for $x$ near $a$.
18. Increments and differentials.

Practice the following problems from an old test:

1. Let $f(x)=\frac{x}{1+x^2}$. Find $f'(2)$.
2. Let $f(x)=(x^3+2x+3)e^{2x}$. Find $f'(0)$.
3. Find an equation of the tangent line to the curve $y=\sqrt[3]{2x^2+4}$ at the point $(\sqrt{2},2)$.
4. Evaluate the limit $\lim_{\theta\to 0}\frac{\sin\theta}{2\theta+\tan\theta}$.
5. Find all points on the graph of the function $f(x)=\sin^2 x+3\cos x$ at which the tangent line is horizontal.
6. Use the implicit differentiation to find $\frac{dy}{dx}$ for the curve $2^x+y^2=xy$.
7. Let $f(x)=\sin^{-1}(2x-1)+\tan^{-1}(x^2)$. Find $f'(\frac{1}{2})$.
8. Let $f(x)=\sec x$. Find $f''(\frac{\pi}{6})$.
9. Let $f(x)=\tan^{-1}(2x)+\sin^{-1}(\sqrt{x}\,)$. Find $f'(x)$, but do not simplify.
10. Evaluate the limit $\lim_{n\to\infty}\left(\frac{n+2}{n}\right)^n$.
11. Solve the equation $\cosh x-\sinh x=3$.
12. Let $y=x^{2x}$. Find $\left.\frac{dy}{dx}\right|_{x=e}$.
13. Two cars start moving from the same point. One travels north at 30 mi/h and the other travels east at 40 mi/h. At what rate is the distance between the cars increasing 30 minutes later?
14. Let $y=\log_2|7-3x|$. Find $dy$.

Answers: 1. $-\frac{3}{25}$; 2. 8; 3. $y=\frac{\sqrt{2}}{3}x+\frac{4}{3}$; 4. $\frac{1}{3}$; 5. $x=n\pi$, $n$ : integer; 6. $\frac{y-2^x\ln 2}{2y-x}$; 7. $\frac{50}{17}$; 8. $\frac{10}{9}\sqrt{3}$. 9. $\frac{1}{1+(2x)^2}\cdot 2+\frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{1}{2\sqrt{x}}$; 10. $e^2$; 11. $-\ln 3$; 12. $4e^{2e}$; 13. 50 mi/hr; 14. $\frac{-3}{(7-3x)\ln 2}\,dx$;

Math 1550-24 Review and old test for Exam (3)

Review the following items:

1. Memorize the derivative formulas.
2. Do all homework problems.
3. Review class notes.
4. Absolute maximum, absolute minimum, local maximum, local minimum.
5. Extreme value theorem.
6. Critical numbers and Fermat's theorem.
7. The closed interval method.
8. Rolle's theorem and the mean value theorem.
9. Increasing/decreasing test.
10. The first derivative test.
11. Concavity test.
12. Point of inflection.
13. The second derivative test.
14. Indeterminate forms: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \times \infty$, $\infty - \infty$, $0^0$, $\infty^0$, $1^\infty$.
15. L'Hospital's rule.
16. Curve sketching.
17. Optimization problems.
18. Newton's method.
19. Anti-derivatives.
20. Sums of $\sum_{k=1}^n k = \frac{n(n+1)}{2}$, $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2}\right]^2$.
21. Definition of area.
22. Riemann sums.

Practice the following problems from an old test:

1. Find the critical numbers of the function $f(x) = x^{1/3}(x+1)$.
2. Find the absolute maximum and absolute minimum values of the function $f(x) = x + \frac{4}{x^2},\ 1 \le x \le 4$.
3. Find the interval(s) on which the function $f(x) = x^2 e^{-x}$ is decreasing.
4. Use the second derivative test to find the local maximum and minimum values of the function $f(x) = x - 2\sin x,\ 0 \le x \le \pi$.
5. State the Extreme Value Theorem.
6. Find point(s) of inflection on the curve $f(x) = xe^{-x}$.
7. A piece of wire 1 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a minimum? You must check that it is really a minimum.
8. Find $f(x)$ satisfying $f'(x) = 3^x,\ f(0) = 0$.

Answers: 1. 0 and $-1/4$; 2. max = 5, min = 3; 3. $(-\infty, 0)$ and $(2, \infty)$; 4. Local min $\frac{\pi}{3} - \sqrt{3}$ at $x = \frac{\pi}{3}$, no local max. 6. $(2, 2e^{-2})$; 7. $\frac{4}{4+\pi}$ m long is bent into a square; 8. $\frac{1}{\ln 3}(3^x - 1)$;

Math 1550-24 Review and old test for Exam (4)

Review the following items:

1. Memorize THE 12 formulas, do homework problems, and review class notes.
2. The Fundamental Theorem of Calculus: Part 1 and Part 2.
3. Indefinite integrals.
4. Substitution rule.
5. Even and odd functions.
6. Areas between curves.
7. Method of disks.
8. Method of washers.
9. Method of slicing.
10. Method of cylindrical shells.
11. Work.
12. Average value of a function

Practice the following problems from an old test:

1. Let $f(x) = \int_{-1}^{2x+1} \sqrt{1+t^2}\, dt$. Find $f'(1)$.
2. Evaluate the integral $\int_0^1 x\big(\sqrt[3]{x} - \sqrt{x}\big)\, dx$.
3. Evaluate the integral $\int \big(\sqrt{x} - \frac{1}{\sqrt{x}}\big)^2\, dx$.
4. Evaluate the integral $\int_0^{\ln 2} e^{-x}\, dx$.
5. Evaluate the integral $\int_0^{\pi/4} \sec x \tan x\, dx$.
6. Evaluate the integral $\int \cot^2 x\, dx$.
7. Evaluate the integral $\int \big(\frac{1}{\sqrt{2-x^2}} + \frac{1}{5+x^2}\big)\, dx$.
8. Evaluate the integral $\int x^2\big(x^3+2\big)^5\, dx$.
9. Evaluate the integral $\int_1^e \frac{\ln x}{x}\, dx$.
10. Find the area of the region bounded by $y = e^x$, $y = \cos x$, and $x = \pi/2$.
11. Evaluate the integral $\int_{-1}^1 \left(x^4 + \frac{x^5}{\sqrt{1+x^2}}\right) dx$.
12. Revolve the region bounded by $y = x^2$, $y = 0$, $x = 1$ about the $x$-axis. Find the volume of the resulting solid.
13. Use the method of disks to find the volume of a sphere of radius $r$.
14. Let $R$ be the region bounded by $y = e^x$, $y = 1$, and $x = 2$. Revolve $R$ about the line $x = 3$. Use the method of cylindrical shells to set up an integral (but do not evaluate) for the volume of the resulting solid.
15. An aquarium 20 ft long, 10 ft wide, and 12 ft deep is half full of water. Find the work required to empty the aquarium by pumping all of the water to the top of the aquarium. (Note: the water density is 62.5 lb/ft$^3$.)
16. Find the average value of the function $f(x) = \frac{10}{4+x^2}$ on $[0, 2]$.

Some answers: 1. $2\sqrt{10}$; 2. $1/35$; 3. $\frac{1}{2}x^2 - 2x + \ln x + C$; 4. $1/2$; 5. $\sqrt{2} - 1$; 6. $-\cot x - x + C$; 7. $\sin^{-1}\frac{x}{\sqrt{2}} + \frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + C$.

Math 1550-24 Sample problems for final exam

1. Find $\lim_{x\to -2} \frac{x^2-4}{x^2+x-2}$.
2. Find $\lim_{x\to 0} \left(\frac{1}{x} - \frac{2}{x(2+x)}\right)$.
3. Find the slope of the tangent line to the graph of $y = \frac{1}{1+x} + \sin^{-1} x$ at the point $P(0,1)$.
4. Find the area of the region bounded by the graph of $y = \sin 2x,\ 0 \le x \le \frac{\pi}{2}$, and the $x$-axis.
5. Let $f(x) = \frac{1+x}{1-x}$. Find $f'(-2)$.
6. Find $\int_{-1}^{0} \frac{x}{1+x^4}\, dx$.
7. Let $f(x) = \tan^{-1}(2x)$. Find $f'(1)$.
8. Find the absolute maximum of the function $f(x) = x^3 + 3x^2, -1 \le x \le \frac{1}{2}$.
9. Find $\int_0^1 \frac{1}{\sqrt{2-x^2}}\, dx$.
10. Find the slope of the tangent line to the curve $2x + y^3 = xy$ at the point $P(-1,1)$.
11. Find $D_x^{99} \cos x$.
12. Let $f(x) = 2^x + x^2$. Find $f'(1)$.
13. Find the limit $\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{x^2}$.
14. Find the interval(s) on which the function $f(x) = x^3 e^{-x}$ is increasing.
15. Let $R$ be the region bounded by $y = x^2$, $x = 2$, and the $x$-axis. Find the volume of the solid generated by revolving $R$ about the $x$-axis.
16. Find the area of the region under the graph of $y = \frac{e^x}{1+e^{2x}},\ 0 \le x \le \frac{1}{2}\ln 3$.
17. A hot-air balloon rises vertically at a rate of 2 m/sec. An observer stands 100 meters from a point on the ground directly below the balloon. Find the rate at which the angle of elevation is changing when the balloon is 50 meters above the ground.
18. Sketch the graphs of $\sin^{-1} x$, $\cos^{-1} x$, and $\tan^{-1} x$. Indicate the domains and the ranges.