1. Evaluate the limit $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{x(1+x)}\right]$.

2. Suppose a function $f(x)$ is defined by $f(x)=\begin{cases} ax+7, & \text{if } x<2, \\ 5, & \text{if } x=2, \\ x^2+bx, & \text{if } x>2. \end{cases}$

   Find the values of a and b so that $f(x)$ is continuous at $x=2$.

3. Find the vertical asymptote(s) of the curve $y=\frac{x^3+1}{x^3-4x}$.

4. Evaluate the limit $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-x\right)$.

5. Find the horizontal asymptote(s) of the curve $y=\frac{|x|}{x+1}$.

6. Use the definition of derivative to find $f'(a)$ for the function $f(x)=x^2-3x$. Here a is any fixed real number.

7. Find the x-coordinates of the points on the curve $y=\frac{1}{3}x^3-\frac{1}{2}x^2-x+1$ where the tangent is horizontal.

8. Evaluate the limit $\lim_{x\to 0}\frac{1-\cos x}{x^2}$.

9. Let $f(x)=(x^2+3x)e^x$. Find $f'(0)$.

10. Find an equation of the tangent line to the curve $y=\frac{x+1}{x-1}$ at the point $(2,3)$.

11. Let $f(x)=\sin^4 x-\cos^4 x$. Find $f'(x)$.

12. Let $y=\sqrt{x^2-3x}$. Find $\left.\frac{dy}{dx}\right|_{x=4}$.

13. Use the implicit differentiation to find $\frac{dy}{dx}$ for the curve $x^2+y^2=xy+1$.

14. Let $f(x)=\tan x$. Find $f''(\frac{\pi}{3})$.

15. Let f be a function defined by $f(x)=\begin{cases} x^2, & x\geq 0, \\ x^3, & x<0. \end{cases}$ Check whether the function f is differentiable at 0 and, if so, find $f'(0)$.

16. Let $f(x)=\tan^{-1}(2x)+\sin^{-1}(\sqrt{x})$. Find $f'(x)$.

17. Evaluate the limit $\lim_{n\to\infty}\left(\frac{n+2}{n}\right)^n$.

18. Solve the equation $\cosh x-\sinh x=3$.

19. Let $y=x^{2x}$. Find $\left.\frac{dy}{dx}\right|_{x=e}$.

20. Find the arc length of the curve $y=\frac{2}{3}(x-1)^{3/2}$, $1\leq x\leq 4$.

21. Let $y = \log_2 |7 - 3x|$. Find dy.

22. Two cars start moving from the same point. One travels north at 30 mi/h and the other travels east at 40 mi/h. At what rate is the distance between the cars increasing 30 minutes later?

23. Find the critical numbers of the function $f(x) = x^{1/3}(x+1)$.

24. Find the absolute maximum and minimum of the function $f(x) = x + \dfrac{4}{x^2}$, $1 \le x \le 4$.

25. Find the intervals on which the function $f(x) = x^2 e^{-x}$ is decreasing.

26. Use the second derivative test to find the local maximum and minimum values of the function $f(x) = x - 2\sin x$, $0 \le x \le \pi$.

27. Evaluate the limit $\lim\limits_{x\to 0^+} x \ln x$.

28. Suppose the derivative $f'(x)$ of a function $f(x)$ is given by $f'(x) = (x+3)e^{-x}$. Check that there is exactly one inflection point in the graph of $y = f(x)$ and find its x-coordinate.

29. Express the limit $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} (3(x_i^*)^2 - 5x_i^*)\Delta x$ as a definite integral on the interval $[0, 2]$.

30. Let $f(x) = \displaystyle\int_{-1}^{x} \sqrt{1+t^2}\, dt$. Find $f'(1)$.

31. Find the antiderivative $F(x)$ of $f(x) = 3 - 2(1+x^2)^{-1}$ satisfying $F(1) = 3$.

32. Let $g(x) = \displaystyle\int_0^{\tan x} \frac{1}{\sqrt{1+t^2}}\, dt$, $0 \le x \le \pi/2$. Find $g'(x)$.

33. Evaluate the integral $\displaystyle\int_0^{\ln 2} e^{2x}\, dx$.

34. Evaluate the integral $\displaystyle\int_0^{\pi/4} \sec x \tan x\, dx$.

35. Evaluate the integral $\displaystyle\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$.

36. Evaluate the integral $\displaystyle\int x^2 (x^3+2)^5\, dx$.

37. Evaluate the integral $\displaystyle\int_1^e \frac{\ln x}{x}\, dx$.

38. Find the area of the region bounded by $y = e^x$, $y = \cos x$, and $x = \pi/2$.

39. Evaluate the integral $\displaystyle\int_{-1}^{1} \left(x^4 + \frac{x^5}{\sqrt{1+x^2}}\right) dx$.

40. Revolve the region bounded by $y = x^2$, $y = 0$, $x = 1$ about the x-axis. Find the volume of the resulting solid.

41. Use the method of disks to find the volume of a sphere of radius r.

42. Let R be the region bounded by $y = e^x$, $y = 1$, and $x = 2$. Revolve R about the line $x = 3$. Use the method of cylindrical shells to set up an integral (but do not evaluate) for the volume of the resulting solid.

43. An aquarium 20 ft long, 10 ft wide, and 12 ft deep is half full of water. Find the work required to empty the aquarium by pumping all of the water to the top of the aquarium. (Note: the water density is 62.5 lb/ft^3.)

44. Find the arc length of the curve $y = \cosh x$, $0 \le x \le 3$.

45. Evaluate the limit $\displaystyle\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{x\sqrt{1+x}}\right)$.

46. Evaluate the limit $\displaystyle\lim_{x\to 7^-}\left([\![x]\!] + [\![5-x]\!]\right)$.

47. Suppose a function $f(x)$ is defined by $f(x) = \begin{cases} x+2, & \text{if } x < 3, \\ a, & \text{if } x = 3, \\ x^2 + bx, & \text{if } x > 3. \end{cases}$

Find the values of a and b so that $f(x)$ is continuous on $(-\infty, \infty)$.

48. Find the vertical asymptote(s) of the curve $y = \dfrac{x^3 - 1}{x^4 - 4x^2}$.

49. Evaluate the limit $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+x} - \sqrt{x^2+1}\right)$.

50. Find the horizontal asymptote(s) of the curve $\dfrac{1-|x|}{1+2x}$.

51. Evaluate the limit $\displaystyle\lim_{x\to\infty}\frac{1-\sqrt{x}}{1+\sqrt{x}}$.

52. Use the definition of derivative to find $f'(a)$ for the function $f(x) = 3 + \sqrt{x}$. Here a is a fixed positive number.

53. Find the x-coordinates of the points on the curve $f(x) = x^3 - 3x^2 + 25$ where the tangent line has a slope 6.

54. Let $f(x) = \dfrac{x}{1+x^2}$. Find $f'(2)$.

55. Let $f(x) = (x^3 + 2x + 3)e^{2x}$. Find $f'(0)$.

56. The position function of a particle is given by $s = t^3 + t^2 - 5t$, $t \ge 0$. When does the particle reach a velocity of 3 m/sec?

57. Find an equation of the tangent line to the curve $y = \sqrt[3]{2x^2+4}$ at the point $(\sqrt{2}, 2)$.

58. Evaluate the limit $\displaystyle\lim_{\theta\to 0}\frac{\sin\theta}{2\theta + \tan\theta}$.

59. Find all points on the graph of the function $f(x) = \sin^2 x + 3\cos x$ at which the tangent line is horizontal.

60. Use the implicit differentiation to find $\dfrac{dy}{dx}$ for the curve $2^x + y^2 = xy$.

61. Let $f(x) = \sin^{-1}(2x-1) + \tan^{-1}(x^2)$. Find $f'(\frac{1}{2})$.

62. Let $f(x) = \sec x$. Find $f''(\frac{\pi}{6})$.

63. Find the limit $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^{3n}$.

64. Let $y = 3^x - 2x$. Find the y-differential.

65. A kite 75 ft above the ground moves horizontally at a speed of 8 ft/sec. At what rate is the angle between the string and the horizontal decreasing when 150 ft of string have been let out?

66. Find the critical numbers of the function $f(x) = \sinh x + \cosh x - 6x$.

67. Find the critical numbers of the function $f(x) = x^{2/3}(x + 4)$.

68. Find the absolute maximum and minimum values of $f(x) = x^2 + \dfrac{16}{x}, \quad 1 \le x \le 4$.

69. Use the first derivative test to find local maximum and minimum values of the function $f(x) = x^3 + 3x^2 - 9x$.

70. Use the second derivative test to find local maximum and minimum values of the function $f(x) = x - \sqrt{2}\,\sin x, \quad 0 \le x \le \pi$.

71. Suppose $f'(x) = (x+1)(x-1)^2(x-3)e^{4x}$. Find the intervals on which f is increasing.

72. Suppose $f''(x) = (2 - \ln x)(x - 1)$. Find the intervals on which f is concave upward.

73. Find point(s) of inflection on the curve $f(x) = xe^{-x}$.

74. A piece of wire 1 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a minimum?

75. Let $f(x) = \displaystyle\int_{-1}^{2x+1} \sqrt{1 + t^2}\, dt$. Find $f'(1)$.

76. Evaluate the integral $\displaystyle\int_0^1 x(\sqrt[3]{x} - \sqrt{x})\, dx$.

77. Evaluate the integral $\displaystyle\int_0^{\ln 2} e^{-x}\, dx$.

78. Evaluate the integral $\displaystyle\int \cot^2 x\, dx$.

79. Evaluate the integral $\displaystyle\int \left(\frac{1}{\sqrt{2 - x^2}} + \frac{1}{5 + x^2}\right) dx$.

80. Evaluate the integral $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\, dx$.

81. Find the area of the region bounded by $y = e^x$ and $y = \sin x$ for $0 \le x \le \pi$.

82. Evaluate the integral $\displaystyle\int_{-2}^{2} \left(|x| + \frac{x^7}{\sqrt{1 + x^4}}\right) dx$.

83. Let R be the region bounded by $y = e^x$, $y = 1$, and $x = 2$. Revolve R about the line $x = -5$. Use the method of cylindrical shells to set up an integral (but do not evaluate) for the volume of the resulting solid.

84. Find the average value of the function $f(x) = \dfrac{1}{3+x^2}$, $0 \le x \le 3$.

85. Find the arc length of the curve $y = \dfrac{2}{3}x^{3/2}$, $0 \le x \le 3$.

86. Find the arc length of the curve $y = \ln \sec x$, $0 \le x \le \pi/4$.

87. Evaluate the limit $\displaystyle\lim_{t \to 4} \frac{4-t}{2-\sqrt{t}}$

88. Find the horizontal asymptotes of the curve $y = \dfrac{e^x+1}{e^x-1}$.

89. Evaluate the limit $\displaystyle\lim_{x \to \infty} \left(\sqrt{x^2+x} - x\right)$.

90. Determine whether or not $f'(0)$ exists for the function

$$f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & \text{if } x \ne 0, \\ 0, & \text{if } x = 0. \end{cases}$$

91. Find the x-coordinates of the points on the curve $y = \frac{1}{3}x^3 - x^2 - 2x + 5$ where the tangent is horizontal.

92. Let $y = x^2 f(x)$. Suppose $f(2) = -1$ and $f'(2) = 3$. Find $\left.\dfrac{dy}{dx}\right|_{x=2}$.

93. Find an equation of the tangent line to $y = \dfrac{x}{x+1} + 2$ at the point where $x = 1$.

94. Evaluate the limit: (a) $\displaystyle\lim_{\theta \to 0} \frac{\theta}{\tan\theta}$, (b) $\displaystyle\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$.

95. Differentiate the function, but do not simplify:

(a) $f(x) = e^{2x}\sin(3x)$, (b) $f(x) = \sin^{-1}(x^2)$, (c) $f(x) = \sqrt{x+\sqrt{x}}$.

96. Find the slope of the tangent line to the curve $x^3 + y^3 = 3xy$ at the point $(\frac{2}{3}, \frac{4}{3})$.

97. Let $y = x\sqrt{x}$. Find $\left.\dfrac{d^3y}{dx^3}\right|_{x=1}$.

98. Let $f(x) = \ln|2x - x^3|$. Find $f'(-1)$.

99. Let $y = x^{2x}$. Fin $\dfrac{dy}{dx}$.

100. Find the critical numbers of the function $f(x) = |x| - |2x+3|$.

101. The altitude of a triangle is increasing at a rate of 2 cm/sec while the area of the triangle is decreasing at a rate of 5 cm^2/sec. At what rate is the base of the triangle changing when the altitude is 3 cm and the area is 15 cm^2?

102. Find the absolute maximum and absolute minimum values of the function

$$f(x) = x + \frac{2}{x}, \quad \frac{1}{2} \le x \le 2.$$

103. Let $f(x) = x - \sqrt{2}\sin x$, $0 \le x \le \pi$. Find the interval(s) on which the fufnction f is increasing.

104. Let $f(x) = x^4 - 2x^3$. Find the interval(s) on which the graph of f is concave upward.

105. If $\sinh x = \frac{1}{2}$. Find $\cosh x$.

106. Evaluate the limit $\lim\limits_{x\to\infty} (x^2 + 5x)e^{-x}$.

107. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2 + 9y^2 = 1$.

108. Given that the graph of f passes through the point $(0, 5)$ and that the slope of its tangent line at $(x, f(x))$ is $\cos x + \sin x$, find $f(\pi/2)$.

109. Let $f(x) = \displaystyle\int_1^{2x} \sqrt{1 + t + t^2}\, dt$. Find $f'(1)$.

110. Evaluate the integral $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}\, dx$.

111. Let $f(x) = \begin{cases} e^x, & \text{if } x \geq 0; \\ 1, & \text{if } x < 0. \end{cases}$ Find $\displaystyle\int_{-1}^{2} f(x)\, dx$.

112. Evaluate the integrals: (a) $\displaystyle\int_0^{1/2} \frac{1}{\sqrt{1 - x^2}}\, dx$. (b) $\displaystyle\int_0^{\sqrt{6}} \frac{1}{2 + x^2}\, dx$.

113. Evaluate the integral $\displaystyle\int_{-2}^{2} \left(x^2 + x\sqrt{x^4 + x^2 + 1}\right) dx$.

114. Evaluate the integrals: (a) $\displaystyle\int \left(x - \frac{1}{x}\right)^2 dx$, (b) $\displaystyle\int \sec x(\sec x + \tan x)\, dx$.

115. Evaluate the limit $\lim\limits_{n\to\infty} \dfrac{1}{n}\left(\sqrt[3]{\dfrac{1}{n}} + \sqrt[3]{\dfrac{2}{n}} + \cdots + \sqrt[3]{\dfrac{n}{n}}\right)$.

116. Evaluate the integral $\displaystyle\int \frac{1 + x}{1 + x^2}\, dx$.

117. Find the area of the region enclosed by the curves $y = x^2$ and $y = x + 2$.

118. The base of a solid is a circular disk with radius r. Parallel cross-sections perpendicular to the base are squares. Find the volume of the solid.

119. Rotate the region enclosed by the curves $y^2 = x$ and $x = 2y$ about the line $x = -1$. Set up two integrals, one in x and another in y, to find the volume of this solid of revolution.

120. A cable that weighs 2 lb/ft is used to lift 1,000 lb of coal up a mineshaft 600 ft deep. Find the work done.

121. Find the arc length of the curve $y = \dfrac{1}{6}x^3 + \dfrac{1}{2x}$, $1 \leq x \leq 2$.

122. Verify the formula $A = 4\pi r^2$ for the surface area of a sphere of radius r.

123. Rotate $y = x^2$, $0 \leq x \leq 1$, about the y-axis. Find the area of the resulting surface.