Coverage of Math 7311 (Fall 2001)

I. Metric spaces

1. metric and examples
2. ℓ^p for $0 < p < \infty$
3. $C[a, b]$
4. Hölder inequality
5. Young inequality
6. Minkowski inequality
7. Topology induced by a metric

II. Complete metric spaces

1. Cauchy sequence
2. Convergent sequence
3. Completeness
4. The nested sphere theorem
5. Denseness
6. Nowhere dense
7. First and second categories
8. The Baire category theorem
9. Cantor set
10. Completion of a metric space

III. Compactness in metric spaces

1. Bounded set
2. Diameter of a set
3. Totally bounded set
4. Hilbert cube
5. Necessary and sufficient condition for compactness
6. Precompact (relatively compact)
7. Heine-Borel theorem
8. Uniformly bounded family of functions
9. Equi-continuous family of functions
10. Pointwise bounded family of functions
11. Arzelà-Ascoli theorem

IV. Functions on metric spaces

1. Continuous function
2. Uniformly continuous function
3. Compactness, continuity, and uniform continuity
4. Upper semi-continuous function
5. Lower semi-continuous function
6. Maximum value of an upper semi-continuous function
7. Minimum value of a lower semi-continuous function

V. Normed linear spaces

1. Norm, normed space
2. Banach space
3. Metric induced by a norm
4. Topology induced by a norm
5. Closed subspace spanned by a set of vectors
6. Completeness of a set of vectors
7. Equivalence of norms

VI. Inner product spaces

1. Inner product, inner product space
2. Schwarz inequality
3. Norm induced by an inner product
4. Hilbert space
5. Orthogonality
6. Orthogonal basis, orthonormal basis
7. Separable Hilbert space
8. Bessel's inequality
9. Parseval's identity
10. Fourier series
11. Riesz-Fischer theorem
12. Isomorphism of Hilbert spaces
13. Orthogonal complement
14. Orthogonal direct sum of a Hilbert space
15. Parallelogram law
16. Characterization of inner product spaces

VII. Measure

1. Riemann integrability
2, Limit and Riemann integrability
3. Measure of modified Cantor set
4. General ideas and construction of measures
5. σ-field and measurable space
6. σ-additivity, measure, and measure space
7. Field and finite additivity
8. σ-additivity on a field
9. Outer and inner measures
10. Measurable sets
11. Lebesgue measure
12. Completeness of a measure
13. Non-measurable set
14. Translation invariance of Lebesgue measure on $\mathbb{R}^n$

VIII. Integration

1. Borel field and Lebesgue field
2. Measurable functions; simple functions
3. Limit and operations on measurable functions
4. Equivalent functions
5. Convergence almost everywhere (Convergence almost surely)
6. Egorov's theorem
7. Riemann integral and Lebesgue integral
8. Three steps in defining the Lebesgue integral
9. Lebesgue integrability
10. Elementary properties of the Lebesgue integral
11. Approximation by simple functions
12. Chebyshev's inequality
13. Monotone convergence theorem
14. Fatou's lemma
15. Lebesgue dominated convergence theorem
16. Convergence in measure (Convergence in probability)
17. Borel-Cantelli lemma
18. Relationship between convergence a.e. and convergence in measure

IX. L^p **spaces**

1. Hölder inequality
2. Minkowski inequality
3. Essentially bounded functions; essential supremum
4. Banach spaces L^p, $1 \le p \le \infty$
5. Metric spaces L^p, $0 < p < 1$
6. Mean convergence
9. Relationship between mean convergence and convergence in measure
10. Relationship between mean convergence and convergence a.e.
11. Denseness of simple functions in L^p spaces, $1 \le p \le \infty$
12. Regular measures
13. Denseness of continuous functions in L^p spaces, $1 \le p < \infty$
14. Separability and non-separability of L^p spaces, $1 \le p \le \infty$
15. Hilbert space L^2

X. Fundamental theorem of calculus for Lebesgue integrals

1. Fundamental theorem of calculus for Riemann integrals
2. Vitali covering and Vitali theorem
3. Differentiation of monotone functions
4. Fundamental theorem of calculus for Lebesgue integrals (part 1)
5. Functions of bounded variation
6. Riemann-Stieljes integrals and Lebesgue integrals
7. Absolutely continuous functions
8. Fundamental theorem of calculus for Lebesgue integrals (part 2)

XI. Continuous linear transformations and dual spaces

1. Continuous linear transformations
2. Continuous linear functionals
3. Operator norm
4. Dual space
5. Riesz representation theorem for H
6. Dual spaces c_0^* and $(\ell^p)^*$
7. Second dual space
8. Gel'fand mapping
9. Reflexivity
10. Riesz representation theorem for L^p
11. Riesz representation theorem for $C[a, b]$
12. Open mapping theorem