Coverage of Math 7360-1 (Fall 2007)

Chapter 0 Review of elementary probability theory

1. Sample space, events
2. Random variables (discrete and continuous)
3. Expectation and variance
4. Conditional expectation
5. Limit theorems (SLLN, WLLN, and CLT)

Chapter 1 Distribution function

1. Monotone function
2. Distribution function
3. Absolutely continuous function
4. Singularly continuous function
5. Decomposition of a distribution function

Chapter 2 Measure theory

1. Field and σ-field
2. Monotone class
3. Monotone class theorem
4. Probability measure
5. Continuity condition
6. Construction of probability measures
7. Distribution function
8. Complete probability space

Chapter 3 Random variable, expectation, and independence

1. Random variable, measurable function
2. Distribution or law of a random variable
3. Lebesgue integral
4. Expectation
5. Change of variables formula
6. Fatou's lemma
7. Monotone convergence theorem
8. Lebesgue dominated convergence theorem
9. Hölder inequality
10. Minkowski inequality
11. Jensen inequality
12. Independence of events
13. Independence of random variables
14. Product measure
15. Kolmogorov's consistency condition
16. Kolmogorov's extension theorem

14. Tightness of a collection of probability measure
15. Lévy equivalence theorem
16. Bochner theorem

Chapter 7 Central limit theorem

1. CLT for iid random variables
2. Triangular array
3. Lindeberg condition
4. Lindeberg–Feller theorem
5. Stable law
6. Stable law by its characteristic function
7. Stable laws as limiting distributions
8. Representation of symmetric stable distributions
9. Representation of stable distributions
10. Infinitely divisible law
11. Infinitely divisible law by its characteristic function
12. Infinitely divisible laws as limiting distributions
13. Lévy-Khinchin representation theorem
14. Lévy three components of an infinitely divisible law
15. Compound Poisson distribution
16. Law of small numbers
17. Law of iterated logarithm

Chapter 9 Conditional expectation and martingales

1. Conditional expectation
2. Radon–Nikodym derivative
3. Properties of conditional expectation
4. Filtration
5. Martingales, supermartingales, submartingales
6. Doob's submartingale inequality
7. Doob's decomposition theorem
8. Uniform integrability
9. Submartingale convergence theorem
10. Martingale convergence theorem