Coverage of Math 7390-2 (Spring 2003)

Chapter 1   Wiener-Itô decomposition theorem
1. Brownian motion
2. Wiener integrals
3. Homogeneous chaos
4. Multiple Wiener integrals
5. Wiener-Itô decomposition theorem
6. Orthogonal basis for $L^2(\Omega)$

Chapter 2   Measures on infinite dimensional spaces
1. Non-existence of Lebesgue measure on ∞-dim spaces
2. Gauss measure on Hilbert space
3. Measurable norms
4. Abstract Wiener spaces
5. Wiener-Itô decomposition
6. Infinite dimensional Bochner theorem
7. Nuclear spaces

Chapter 3   Stochastic integrals
1. Quadratic variation of Brownian motion
2. Stochastic integrals
3. Relationship with Multiple Wiener integrals
4. Martingale representation theorem
5. Itô lemma
6. Stochastic differential equations
7. Properties of solutions
8. Feynman-Kac formula

Chapter 4   Applications to mathematical finance
1. The binomial asset pricing model
2. Market, portfolio, and arbitrage
3. Cameron-Martin-Maruyama theorem
4. Girsanov theorem
5. Non-existence of arbitrage
6. Attainability and completeness
7. Option pricing

Chapter 5   Theory of generalized functions
1. A simple example
2. Test functions and generalized functions
3. Schwartz space and tempered distributions
4. Continuous operators on the Schwartz space
5. Nuclearity of the Schwartz space
6. White noise space
7. White noise as a generalized function