Coverage of Math 7390-3 (Spring 2004)

Chapter 1 Brownian motion

1. Introduction
2. Definition of Brownian motion
3. Simple properties of Brownian motion
4. Construction of Brownian motion
5. Wiener integral
6. Martingales

Chapter 2 Itô integral

1. Two simple examples
2. Stochastic integrals
3. Examples of stochastic integrals
4. Martingales defined by Itô integrals
5. Doob's submartingale inequality
6. Continuity property

Chapter 3 An extension of stochastic integrals

1. A simple example
2. More general integrands
3. A key lemma
4. General stochastic integrals
5. Stopping times
6. Associated stochastic processes

Chapter 4 Stochastic integrals for martingales

1. A simple example
2. Poisson processes
3. A preliminary theorem
4. Martingales as integrators
5. Extension for integrands

Chapter 5 Itô formula

1. Motivation
2. The Itô formula
3. Some implications of the Itô formula
4. Generalizations and applications

Chapter 6 Applications to mathematical finance

1. Market, portfolio, value, and self-financing
2. Admissibility and arbitrage
3. Girsanov theorem
4. Nonexistence of an arbitrage
5. T-claim, attainability, hedging, and completeness
6. Option pricing
7. The Black-Scholes model

Chapter 7 White noise theory

1. Wiener-Itô decomposition theorem
2. What is white noise?
3. White noise space
4. Test and generalized functions
5. S-transform
6. Characterization theorem
7. Some applications