Math 1550 EXAM (1)

1. Evaluate the function $f(x) = \dfrac{\tan(2x)}{x}$ at the numbers $x = 0.01, 0.001, 0.0001$ to guess the value of $\lim\limits_{x\to 0} \dfrac{\tan(2x)}{x}$.
2. Evaluate the limit $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-x-2}$.
3. Evaluate the limit $\lim\limits_{x\to 0} \left[\dfrac{1}{x} - \dfrac{1}{x(1+x)}\right]$.
4. Evaluate the limit $\lim_{x\to 3^+} \Big(\llbracket x \rrbracket + \llbracket -x \rrbracket\Big)$.
5. Suppose a function $f(x)$ is defined by $f(x) = \begin{cases} ax+7, & \text{if } x < 2, \\ 5, & \text{if } x = 2, \\ x^2 + bx, & \text{if } x > 2. \end{cases}$ Find the values of a and b so that $f(x)$ is continuous at $x = 1$.
6. Find the vertical asymptote(s) of the curve $y = \dfrac{x^3+1}{x^3-4x}$.
7. Evaluate the limit $\lim\limits_{x\to\infty} \Big(\sqrt{x^2+3x} - x\Big)$.
8. Find the horizontal asymptote(s) of the curve $y = \dfrac{|x|}{x+1}$.
9. Use the **definition** of derivative to find $f'(1)$ for the function $f(x) = \sqrt{x}$.
10. Use the **definition** of derivative to find $f'(a)$ for the function $f(x) = x^2 - 3x$. Here a is any fixed real number.

Math 1550 EXAM (2)

1. Find the x-coordinates of the points on the curve $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 1$ where the tangent is horizontal.
2. Evaluate the limit $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.
3. Let $f(x) = (x^2+3x)e^x$. Find $f'(0)$.
4. Find an equation of the tangent line to the curve $y = \dfrac{x+1}{x-1}$ at the point $(2,3)$.
5. A spherical balloon is being inflated. Find the rate of increase of the volume ($V = \frac{4}{3}\pi r^3$) with respect to the radius r when $r = 2$ ft.
6. Let $f(x) = \sin^4 x - \cos^4 x$. Find $f'(x)$. (The answer should be in the simplest form.)
7. Let $y = \sqrt{x^2-3x}$. Find $\left.\dfrac{dy}{dx}\right|_{x=4}$.
8. Use the implicit differentiation to find $\dfrac{dy}{dx}$ for the curve $x^2 + y^2 = xy + 1$.
9. Let $f(x) = \tan x$. Find $f''(\frac{\pi}{3})$.
10. Let f be a function defined by $f(x) = \begin{cases} x^2, & x \geq 0, \\ x^3, & x < 0. \end{cases}$ Check whether the function f is differentiable at 0 and, if so, find $f'(0)$.

Bonus Prove that if a function is differentiable at a, then it is continuous at a.

Math 1550 EXAM (3)

1. Let $f(x) = \tan^{-1}(2x) + \sin^{-1}(\sqrt{x}\,)$. Find $f'(x)$, but do not simplify.
2. Evaluate the limit $\lim_{n\to\infty} \left(\frac{n+2}{n}\right)^n$.
3. Solve the equation $\cosh x - \sinh x = 3$.
4. Let $y = x^{2x}$. Find $\left.\frac{dy}{dx}\right|_{x=e}$.
5. Two cars start moving from the same point. One travels north at 30 mi/h and the other travels east at 40 mi/h. At what rate is the distance between the cars increasing 30 minutes later?
6. Let $y = \log_2 |7 - 3x|$. Find dy.
7. Find the critical numbers of the function $f(x) = x^{1/3}(x+1)$.
8. Find the absolute maximum and absolute minimum values of the function $f(x) = x + \frac{4}{x^2},\ 1 \le x \le 4$.
9. Find the interval(s) on which the function $f(x) = x^2 e^{-x}$ is decreasing.
10. Use the second derivative test to find the local maximum and minimum values of the function $f(x) = x - 2\sin x,\ 0 \le x \le \pi$.

Bonus State the Extreme Value Theorem.

Math 1550 EXAM (4)

1. Evaluate the limit $\lim_{x\to 1} \frac{x^{10} - 2x^7 + 1}{x^4 - 1}$.
2. Evaluate the limit $\lim_{x\to 0^+} x \ln x$.
3. Suppose the derivative $f'(x)$ of a function $f(x)$ is given by $f'(x) = (x+3)e^{-x}$. Check that there is exactly one inflection point in the graph of $y = f(x)$ and find its x-coordinate.
4. Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \left(3(x_i^*)^2 - 5x_i^*\right)\Delta x$ as a definite integral on the interval $[0, 2]$.
5. Let $f(x) = \int_{-1}^{x} \sqrt{1+t^2}\,dt$. Find $f'(1)$.
6. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a minimum? You must check that it is really a minimum.
7. Find the antiderivative $F(x)$ of $f(x) = 3 - 2(1+x^2)^{-1}$ satisfying $F(1) = 3$.
8. Evaluate the integral $\int_0^{\sqrt{5}} \sqrt{5 - x^2}\,dx$.
   (Hint: Use the geometric interpretation of a definite integral.)
9. Let $g(x) = \int_0^{\tan x} \frac{1}{\sqrt{1+t^2}}\,dt,\ 0 \le x \le \pi/2$. Find $g'(x)$. The answer must be in the simplest form.
10. Evaluate the integral $\int_0^{\ln 2} e^x\,dx$.

11. Evaluate the integral $\displaystyle\int_0^{\pi/4} \sec x \tan x \, dx$.
12. Evaluate the integral $\displaystyle\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$.

Math 1550 EXAM (5)

1. Evaluate the integral $\displaystyle\int x^2 \left(x^3 + 2\right)^5 dx$.
2. Evaluate the integral $\displaystyle\int_1^e \frac{\ln x}{x} \, dx$.
3. Find the area of the region bounded by $y = e^x$, $y = \cos x$, and $x = \pi/2$.
4. Evaluate the integral $\displaystyle\int_{-1}^1 \left(x^4 + \frac{x^5}{\sqrt{1+x^2}}\right) dx$.
5. Revolve the region bounded by $y = x^2$, $y = 0$, $x = 1$ about the x-axis. Find the volume of the resulting solid.
6. Use the method of disks to find the volume of a sphere of radius r.
7. Let R be the region bounded by $y = e^x$, $y = 1$, and $x = 2$. Revolve R about the line $x = 3$. Use the method of cylindrical shells to set up an integral (but do not evaluate) for the volume of the resulting solid.
8. An aquarium 20 ft long, 10 ft wide, and 12 ft deep is half full of water. Find the work required to empty the aquarium by pumping all of the water to the top of the aquarium. (Note: the water density is 62.5 lb/ft^3.)
9. Find the average value of the function $f(x) = \dfrac{10}{4+x^2}$ on $[0, 2]$.
10. Find the arc length of the curve $y = \dfrac{2}{3}(x-1)^{3/2}$, $1 \le x \le 4$.
11. Find the arc length of the curve $y = \cosh x$, $0 \le x \le 3$.