Math 2030
Spring, 2007

Study Sheet, Test 2

The 2nd test on Friday, April 13, will cover Chapters 9 through 12.

Definitions. You should know basic definitions: the three properties of a metric, dense subset, open subset, closed subset, symbolic dynamics (the sequence space, its metric, and the shift map), know the 3 propeties of a conjugacy and of a semiconjugacy, chaotic dynamical system, (topological) transitivity, dense orbit, sensitive dependence on initial conditions, subshift, basin of attraction, immediate basic of attraction.

Theory. You should know the Proximity Theorem. You should be able to prove that simple functions like the shift map in symbolic dynamics are continuous. You should be able to prove that conjugacies preserve fixed points and periodic points of period 2. You should know the Sarkovskii order on the positive integers and what the Sarkovskii theorem and its converse say. For $Q_c(x)$ with $c < -2$, know how to obtain the itinerary of a point in the stable Cantor set and assign to it a member of Σ, the sequence space; know that this assignment is a conjugacy. You should know the symbolic dynamic system that results from the study of the quadratic $Q(x) = x^2 + c$ for the case of the super-attracting periodic 3 orbit (see Section 11.4). You should know and understand the major dynamical systems theorem having to do with the Schwarzian derivative (p. 158) and understand the role of the critical orbit as expressed in the theorem..

Basic Concepts. Know how to test that a set is dense, particularly for the sequence space Σ. Know the metric for Σ and how to compute distances. Know how to show topological transitivity or dense orbits for symbolic dynamics. Know basic examples of chaotic systems (symbolic dynamics, restricted symbolic dynamics on Σ', $Q_{-2}(x)$, the logistic map $F(x) = 4x(1-x)$, the tent map with constant 2). Know how to check that functions are conjugacies in elementary cases. Know how to start with two positive integers and show which is larger in the Sarkovskii order. Know how to show that simple piecewise linear functions have an orbit of one size, but not another (like several of the problems at the end of Chapter 11). Know how the Feigenbaum constant is defined from an orbit diagram. Know how to interpret graphs as subshifts. Know how to compute the Schwarzian derivative and work with the Schwarzian chain rule. (You need not memorize these, however.)

Homework Problems. You should be able to work problems that are similar to assigned homework problems.