Math 7550, Spring 2006 **QUIZ 2** *Name*:

Do all three problems

1. (4 points) Suppose that $\mathcal{A} = \{(U_i, \phi_i) : i \in I\}$ is an atlas for M. What condition must a chart (U, ϕ) satisfy to be in the differentiable structure generated by this atlas. What must one prove about any two charts satisfying this condition to verify that all such charts do indeed form a differentiable structure?

Answer:
(i) It must be compatible with all charts in the atlas.
(ii) One must prove they are compatible with each other.

2. (3 points) Let M be a smooth manifold with a finite atlas $\{(U_i, \phi_i) : i \in I\}$. How could one modify this atlas to obtain a minimal atlas (one in which no charts could be deleted and still have an atlas). (Hint: This relates to Exercise 2* in the homework.)

Answer: Pick out a subfamily of charts $\{(U_j, \phi_j) : j \in J \subseteq I\}$ such that $\{U_j : j \in J\}$ is a minimal open cover of M.

3. (5 points) Let (U, ϕ) belong to the differentiable structure of a smooth manifold M and let (V, ψ) be a chart, where $V \subseteq U$. If (V, ψ) is compatible with (U, ϕ), argue that (V, ψ) is compatible with all charts in the differentiable structure of M.

Answer: Let (U_i, ϕ_i) be any chart. Then

$$\phi_i \circ \psi^{-1} = (\phi_i \circ \phi^{-1}) \circ (\phi \circ \psi^{-1}),$$

$$\psi \circ \phi_i^{-1} = (\psi \circ phi^{-1}) \circ (\phi \circ \phi_i^{-1}).$$

Both are compositions of C^∞-functions, hence C^∞.