The Fascinating Story Behind Our Mathematics

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Introduction

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Math was interesting: In many ancient cultures (Egypt, Mesopotamia, India, China) mathematics became an independent subject, practiced by scribes and others.
Old Babylon (2000-1700 B.C.)

The Mesopotamian scribes represented numbers using a *place value* system based on *sixty*. For example, the number 742 would be written as II HH (600 + 60 + 60) + II HH (10 + 10 + 1 + 1):

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We could “babble on” about Babylon, but instead we move on to speak about...
Those Incredible Greeks

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- Mathematical “truths” must be proved, not just observed. Mathematics became deductive, not inductive.
- Mathematics begins with basic assumptions, called “axioms,” and definitions and builds on itself to add logical consequences or theorems until a whole theory develops.
Greek Hall of Fame

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He also demonstrated some basic geometric theorems:

\[ \alpha = \beta \]

\[ \alpha = 90^\circ \]
The Pythagoreans

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![Pythagorean theorem diagram](image)

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They also studied musical ratios, irrational ratios (between a side and hypothenuse), and perfect numbers (numbers that equal the sum of their divisors) to name a few.
Best-Selling Author

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Other topics covered include divisibility properties of integers and other basic number theory, regular polyhedra, and an advanced theory of ratios.
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- calculated very good approximation to $\pi$ and a variety of areas and volumes,
- invented important new curves, such as the spiral of Archimedes.
Some “Late-Comers”

**Ptolemy** (100 A.D.) was the greatest of Greek astronomers.

Diophantus (300 A.D.) found solutions of equations that were integers or fractions for wide classes of equations, for example, \(m^2 + n^2 = k^2\). These are called **diophantine equations**. He wrote a book about his results that greatly influenced later number theory.
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The most famous invention of the Indian mathematicians/astronomers is their base 10 numeration system. By 600 A.D. they had nine symbols for the numbers 1 to 9, introduced place value, and had created a symbol, a dot or small circle, to denote an empty place (where we write 0). They also developed methods for doing arithmetic with this system.
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This numbering system was picked up by the Arabs and made its way west as the Hindu-Arabic numbers.
The Birth of Zero

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Early important Indian mathematicians included Aryabhata, Brahmagupta, and Bhaskara, the latter two being among the first to work with negative quantities.
Early Trigonometry

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Chord of an angle
The “Half-Chord” or Sine

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“Half-chord” was (mis)translated into Latin as “sinus,” from which we get “sine,”

$$\sin(\alpha) = \frac{1}{2}\text{chord}(2\alpha).$$
Arab Math (800-1300 A.D.)

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- A large library in Baghdad collected and housed Greek and Indian works together with Arab translations. Euclid’s *Elements* had a huge impact.
- The common Arabic language throughout the empire allowed them to collect and build on one another’s work.
Arabic Contributions

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They also made important contributions to geometry, trigonometry and astronomy, and some work in number theory and combinatorics (including early versions of “Pascal’s Triangle”).
Arabic Arithmetic

Al-Khwārizmī (850 A.D.), from the area that is today Uzbekistan, wrote several influential books. One was an explanation of the decimal place value system for writing numbers and doing arithmetic, which came from India. It was the major source from which Europeans learned the new system.
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Many translations into Latin began with the words “dixit Algorismi” (“so says Al-Khwārizmī”), so the word algorism came to mean the process of computing with Hindu-Arabic numbers. The modern corresponding word is “algorithm,” a “recipe” for solving some mathematical problem.
More on Arabic Algebra

Al-Khwārizmī also wrote a basic book about algebra called "al-jabr w‘all-muqābala,” which technically meant “bone-setting,” but more generally “restoration and compensation.” When this book was translated into Latin “al-jabr” became “algebra," the word we still use today.
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The most famous of the Arabic mathematicians was the Persian Omar Khayyam, who worked on cubic equations. But his fame rested primarily on the fact that he was a talented poet. His poems much later appeared in English in a famous translation by Sir Richard Burton.
Mathematics Moves West

The Italian Renaissance included a burst of mathematical activity as Greek and Arabic mathematics became known. The Arabic mathematicians knew how to solve linear and quadratic equations, but made limited progress on solving cubic (degree 3) equations. The crucial breakthrough was made in Italy in a series of discoveries and an accompanying soap opera of events.

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But there was a “little” problem.
Act III

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(5) Lodovico Ferrari (1522-1565), a student of Cardano’s, showed how to extend Cardano’s techniques to solve all equations of degree 4.
Algebra Comes to France

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(2) No one could figure out how to solve fifth degree equations, but a general theory of polynomials and their roots evolved.
Analytic Geometry

(3) Fermat and Descartes linked algebra and geometry using what we now call “coordinate geometry” or “analytic geometry” (thus we call the coordinate plane the “cartesian plane”). Both Fermat and Descartes used it to study curves by their equations and showed the power of algebra to solve difficult geometric problems.
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for all positive integers \( x, y, z \) and \( n \geq 3 \).
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This assertion was not proved true until 1994 by Andrew Wiles of Princeton.
Modern Mathematics

Today we have only had time to sketch the history of our math up to the time of the rise of modern mathematics (1600-2004). With the discovery of the calculus by Newton and Leibnitz in the 2nd half of the 17th century, a deep and wide mathematical stream has developed that has resulted one of the greatest and most profound human intellectual achievements and is today practiced worldwide.

Suggested Reading:
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