The Fascinating Story Behind Our Mathematics

Jimmie Lawson

Louisiana State University

Introduction

In every civilization that has developed writing we also find evidence for some level of mathematical knowledge.

Introduction

In every civilization that has developed writing we also find evidence for some level of mathematical knowledge.

Math was needed for everyday life: commercial transactions and accounting, government taxes and records, measurement, inheritance.

Introduction

In every civilization that has developed writing we also find evidence for some level of mathematical knowledge.

Math was needed for everyday life: commercial transactions and accounting, government taxes and records, measurement, inheritance.

Math was needed in developing branches of knowledge: astronomy, timekeeping, calendars, construction, surveying, navigation

Introduction

In every civilization that has developed writing we also find evidence for some level of mathematical knowledge.

Math was needed for everyday life: commercial transactions and accounting, government taxes and records, measurement, inheritance.

Math was needed in developing branches of knowledge: astronomy, timekeeping, calendars, construction, surveying, navigation

Math was interesting: In many ancient cultures (Egypt, Mesopotamia, India, China) mathematics became an independent subject, practiced by scribes and others.

Old Babylon (2000-1700 B.C.)

The Mesopotamian scribes represented numbers using a *place value* system based on *sixty*.

Old Babylon (2000-1700 B.C.)

The Mesopotamian scribes represented numbers using a *place value* system based on *sixty*.

For example, the number 742 would be written

► ▼▼$(600 + 60 + 60)$ ►► ▼▼$(10 + 10 + 1 + 1)$.

Old Babylon (2000-1700 B.C.)

The Mesopotamian scribes represented numbers using a *place value* system based on *sixty*.

For example, the number 742 would be written

$$\blacktriangleright \blacktriangledown\blacktriangledown(600 + 60 + 60) \blacktriangleright\blacktriangleright \blacktriangledown\blacktriangledown(10 + 10 + 1 + 1).$$

The Babylonians could do arithmetic, including fractions, solve linear equations, extract square roots, and find large Pythagorean triples of integers:

$$(2291)^2 + (2700)^2 = (3541)^2$$

Old Babylon (2000-1700 B.C.)

The Mesopotamian scribes represented numbers using a *place value* system based on *sixty*.

For example, the number 742 would be written

$$\blacktriangleright \blacktriangledown\blacktriangledown(600+60+60)\ \blacktriangleright\blacktriangleright \blacktriangledown\blacktriangledown(10+10+1+1).$$

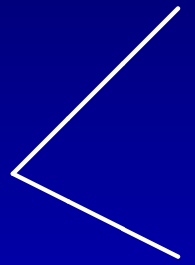
The Babylonians could do arithmetic, including fractions, solve linear equations, extract square roots, and find large Pythagorean triples of integers:

$$(2291)^2 + (2700)^2 = (3541)^2$$

They also studied astronomy (remember the 3 wisemen), which needed elementary trigonometry.

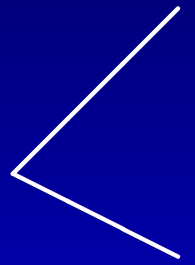
Babylonian Heritage

Our division of hours and degrees into 60 minutes and minutes into 60 seconds is based on Babylonian practice and their base 60 numbering system.



Babylonian Heritage

Our division of hours and degrees into 60 minutes and minutes into 60 seconds is based on Babylonian practice and their base 60 numbering system.



We could “babble on” about Babylon, but instead we move on to speak about...

Those Incredible Greeks

Greek mathematical activity lasted for about 1000 years (600 B.C.-400 A.D.). Of all ancient cultures, the Greeks were by far the most influential on our mathematics and made major contributions, supremely in geometry, but also in number theory.

Those Incredible Greeks

Greek mathematical activity lasted for about 1000 years (600 B.C.-400 A.D.). Of all ancient cultures, the Greeks were by far the most influential on our mathematics and made major contributions, supremely in geometry, but also in number theory.

- Mathematical “truths” must be proved, not just observed. Mathematics became deductive, not inductive.

Those Incredible Greeks

Greek mathematical activity lasted for about 1000 years (600 B.C.-400 A.D.). Of all ancient cultures, the Greeks were by far the most influential on our mathematics and made major contributions, supremely in geometry, but also in number theory.

- Mathematical "truths" must be proved, not just observed. Mathematics became deductive, not inductive.
- Mathematics begins with basic assumptions, called "axioms," and definitions and builds on itself to add logical consequences or theorems until a whole theory develops.

Greek Hall of Fame

Thales was the grandfather of Greek mathematics. He was from the Greek colony of Ionia (western Asia Minor, modern Turkey) and lived about 600 B.C.

Greek Hall of Fame

Thales was the grandfather of Greek mathematics. He was from the Greek colony of Ionia (western Asia Minor, modern Turkey) and lived about 600 B.C.

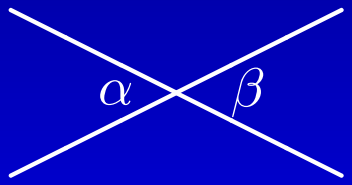
He taught that proofs are the only reliable basis for mathematical knowledge, not observation or measurement.

Greek Hall of Fame

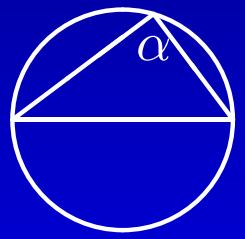
Thales was the grandfather of Greek mathematics. He was from the Greek colony of Ionia (western Asia Minor, modern Turkey) and lived about 600 B.C.

He taught that proofs are the only reliable basis for mathematical knowledge, not observation or measurement.

He also demonstrated some basic geometric theorems:



$\alpha = \beta$



$\alpha = 90^\circ$

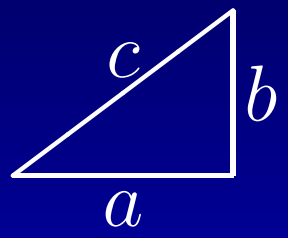
The Pythagoreans

Pythagoras and his school flourished about 500 B.C. in a Greek colony in southeast Italy. They were something of a secret society. Motto: Everything is number.

The Pythagoreans

Pythagoras and his school flourished about 500 B.C. in a Greek colony in southeast Italy. They were something of a secret society. Motto: Everything is number.

The Pythagorean school is known for the Pythagorean theorem (the world's most famous theorem).

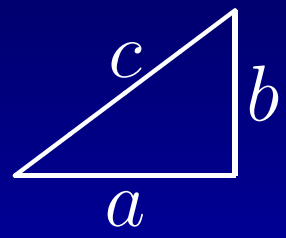


$$a^2 + b^2 = c^2$$

The Pythagoreans

Pythagoras and his school flourished about 500 B.C. in a Greek colony in southeast Italy. They were something of a secret society. Motto: Everything is number.

The Pythagorean school is known for the Pythagorean theorem (the world's most famous theorem).



$$a^2 + b^2 = c^2$$

They also studied musical ratios, irrational ratios (between a side and hypothenuse), and perfect numbers (numbers that equal the sum of their divisors) to name a few.

Best-Selling Author

Euclid lived around 300 B.C. in Alexandria, Egypt. In his book, the *Elements*, he summarized 300 years of Greek mathematics as a systematic and logical theory. Its style and content were enormously influential on later mathematical tradition, the most influential and widely read math book ever written.

Best-Selling Author

Euclid lived around 300 B.C. in Alexandria, Egypt. In his book, the *Elements*, he summarized 300 years of Greek mathematics as a systematic and logical theory. Its style and content were enormously influential on later mathematical tradition, the most influential and widely read math book ever written.

The first part of the book starts with basic axioms and definitions and develops a comprehensive theory of geometry, plane and solid geometry.

Best-Selling Author

Euclid lived around 300 B.C. in Alexandria, Egypt. In his book, the *Elements*, he summarized 300 years of Greek mathematics as a systematic and logical theory. Its style and content were enormously influential on later mathematical tradition, the most influential and widely read math book ever written.

The first part of the book starts with basic axioms and definitions and develops a comprehensive theory of geometry, plane and solid geometry.

Other topics covered include divisibility properties of integers and other basic number theory, regular polyhedra, and an advanced theory of ratios.

The Sand Reckoner

Archimedes (250 B.C.) lived in Sicily and was the greatest mathematician of the ancient world. Some of his achievements were

The Sand Reckoner

Archimedes (250 B.C.) lived in Sicily and was the greatest mathematician of the ancient world. Some of his achievements were

- Combined mathematics and mechanics (principle of lever, various laws about physics of water),

The Sand Reckoner

Archimedes (250 B.C.) lived in Sicily and was the greatest mathematician of the ancient world. Some of his achievements were

- Combined mathematics and mechanics (principle of lever, various laws about physics of water),
- developed method for writing and using very large numbers (calculated how many grains of sand would fill the universe),

The Sand Reckoner

Archimedes (250 B.C.) lived in Sicily and was the greatest mathematician of the ancient world. Some of his achievements were

- Combined mathematics and mechanics (principle of lever, various laws about physics of water),
- developed method for writing and using very large numbers (calculated how many grains of sand would fill the universe),
- calculated very good approximation to π and a variety of areas and volumes,

The Sand Reckoner

Archimedes (250 B.C.) lived in Sicily and was the greatest mathematician of the ancient world. Some of his achievements were

- Combined mathematics and mechanics (principle of lever, various laws about physics of water),
- developed method for writing and using very large numbers (calculated how many grains of sand would fill the universe),
- calculated very good approximation to π and a variety of areas and volumes,
- invented important new curves, such as the spiral of Archimedes.

Some “Late-Comers”

Ptolemy (100 A.D.) was the greatest of Greek astronomers.

Some “Late-Comers”

Ptolemy (100 A.D.) was the greatest of Greek astronomers.

- He wrote the most important book on astronomy until Copernicus. It was widely known in Asia under the title “The Greatest.”

Some “Late-Comers”

Ptolemy (100 A.D.) was the greatest of Greek astronomers.

- He wrote the most important book on astronomy until Copernicus. It was widely known in Asia under the title “The Greatest.”
- Made basic contributions to spherical geometry.

Some "Late-Comers"

Ptolemy (100 A.D.) was the greatest of Greek astronomers.

- He wrote the most important book on astronomy until Copernicus. It was widely known in Asia under the title "The Greatest."
- Made basic contributions to spherical geometry.

Diophantus (300 A.D.) found solutions of equations that were integers or fractions for wide classes of equations, for example, $m^2 + n^2 = k^2$. These are called diophantine equations. He wrote a book about his results that greatly influenced later number theory.

India (500-1200 A.D.)

The most famous invention of the Indian mathematicians/astronomers is their base 10 numeration system. By 600 A.D. they had nine symbols for the numbers 1 to 9, introduced place value, and had created a symbol, a dot or small circle, to denote an empty place (where we write 0). They also developed methods for doing arithmetic with this system.

India (500-1200 A.D.)

The most famous invention of the Indian mathematicians/astronomers is their base 10 numeration system. By 600 A.D. they had nine symbols for the numbers 1 to 9, introduced place value, and had created a symbol, a dot or small circle, to denote an empty place (where we write 0). They also developed methods for doing arithmetic with this system.

$$747 = 7 \times 100 + 4 \times 10 + 7 \times 1.$$

India (500-1200 A.D.)

The most famous invention of the Indian mathematicians/astronomers is their base 10 numeration system. By 600 A.D. they had nine symbols for the numbers 1 to 9, introduced place value, and had created a symbol, a dot or small circle, to denote an empty place (where we write 0). They also developed methods for doing arithmetic with this system.

$$747 = 7 \times 100 + 4 \times 10 + 7 \times 1.$$

This numbering system was picked up by the Arabs and made its way west as the Hindu-Arabic numbers.

The Birth of Zero

The Hindus were the first to think of 0 not only as a place holder (as in 703), but as a number in its own right, with arithmetic properties such as $2 + 0 = 2$ and $7 \times 0 = 0$.

The Birth of Zero

The Hindus were the first to think of 0 not only as a place holder (as in 703), but as a number in its own right, with arithmetic properties such as $2 + 0 = 2$ and $7 \times 0 = 0$.

This required a whole new way of thinking: zero was promoted from being nothing to being an actual number that one could work with.

The Birth of Zero

The Hindus were the first to think of 0 not only as a place holder (as in 703), but as a number in its own right, with arithmetic properties such as $2 + 0 = 2$ and $7 \times 0 = 0$.
This required a whole new way of thinking: zero was promoted from being nothing to being an actual number that one could work with.

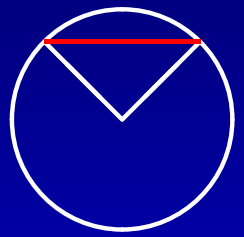
Early important Indian mathematicians included Aryabhata, Brahmagupta, and Bhaskara, the latter two being among the first to work with negative quantities.

Early Trigonometry

Greek and Babylonian astronomers had introduced trigonometry, but the Indians made important contributions. Greek trigonometric calculations had revolved around the *chord* of an angle and chord tables had been compiled.

Early Trigonometry

Greek and Babylonian astronomers had introduced trigonometry, but the Indians made important contributions. Greek trigonometric calculations had revolved around the *chord* of an angle and chord tables had been compiled.



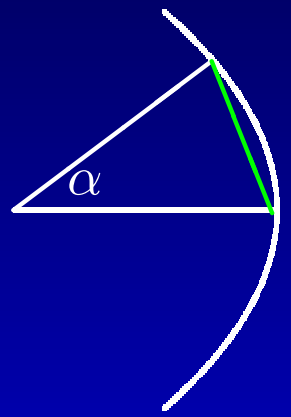
Chord of an angle

The “Half-Chord” or Sine

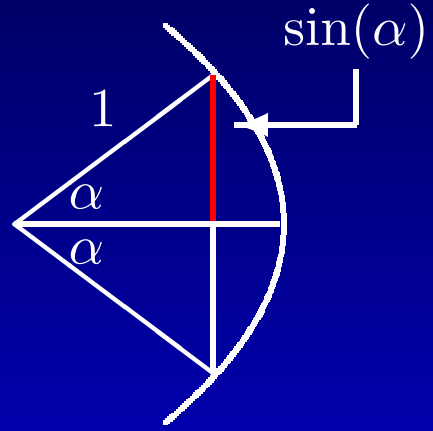
The Indian mathematicians realized the importance of starting with a central angle α, doubling it (2α), and then taking *half* the chord of 2α, which they called the “half-chord.”

The “Half-Chord” or Sine

The Indian mathematicians realized the importance of starting with a central angle α, doubling it (2α), and then taking *half* the chord of 2α, which they called the “half-chord.”



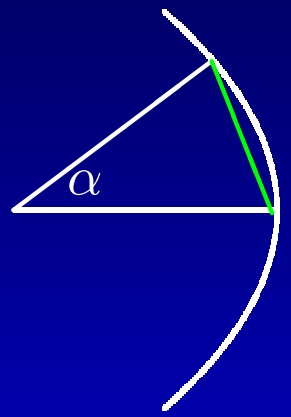
Chord (α)

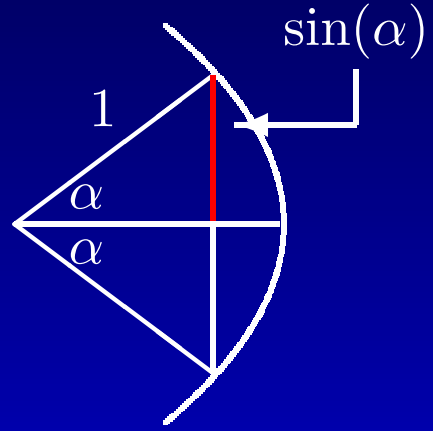


Half-Chord(α)

The “Half-Chord” or Sine

The Indian mathematicians realized the importance of starting with a central angle α, doubling it (2α), and then taking *half* the chord of 2α, which they called the “half-chord.”





Chord (α) Half-Chord(α)

“Half-chord” was (mis)translated into Latin as “sinus,” from which we get “sine,”

$$\sin(\alpha) = (1/2)\text{chord}(2\alpha).$$

Arab Math (800-1300 A.D.)

- By 750 A.D. the Islamic Empire stretched all the way from western India to parts of Spain. Its imperial capital, Baghdad, quickly became the cultural center of the empire.

Arab Math (800-1300 A.D.)

- By 750 A.D. the Islamic Empire stretched all the way from western India to parts of Spain. Its imperial capital, Baghdad, quickly became the cultural center of the empire.
- A large library in Baghdad collected and housed Greek and Indian works together with Arab translations. Euclid's *Elements* had a huge impact.

Arab Math (800-1300 A.D.)

- By 750 A.D. the Islamic Empire stretched all the way from western India to parts of Spain. Its imperial capital, Baghdad, quickly became the cultural center of the empire.
- A large library in Baghdad collected and housed Greek and Indian works together with Arab translations. Euclid's *Elements* had a huge impact.
- The common Arabic language throughout the empire allowed them to collect and build on one another's work.

Arabic Contributions

The development of algebra was the most important part of Arabic mathematics. Arabic mathematicians learned how to manipulate polynomials, solve various algebraic expressions, and much more.

Arabic Contributions

The development of algebra was the most important part of Arabic mathematics. Arabic mathematicians learned how to manipulate polynomials, solve various algebraic expressions, and much more.

All of this was done entirely with words, with no symbols at all. Their methods of solution were frequently geometric.

Arabic Contributions

The development of algebra was the most important part of Arabic mathematics. Arabic mathematicians learned how to manipulate polynomials, solve various algebraic expressions, and much more.

All of this was done entirely with words, with no symbols at all. Their methods of solution were frequently geometric.

They also made important contributions to geometry, trigonometry and astronomy, and some work in number theory and combinatorics (including early versions of “Pascal’s Triangle”).

Arabic Arithmetic

Al-Khwārizmī (850 A.D.), from the area that is today Uzbekistan, wrote several influential books. One was an explanation of the decimal place value system for writing numbers and doing arithmetic, which came from India. It was the major source from which Europeans learned the new system.

Arabic Arithmetic

Al-Khwārizmī (850 A.D.), from the area that is today Uzbekistan, wrote several influential books. One was an explanation of the decimal place value system for writing numbers and doing arithmetic, which came from India. It was the major source from which Europeans learned the new system.

Many translations into Latin began with the words "dixit Algorismi" ("so says Al-Khwārizmī"), so the word *algorism* came to mean the process of of computing with Hindu-Arabic numbers. The modern corresponding word is "algorithm," a "recipe" for solving some mathematical problem.

More on Arabic Algebra

Al-Khwārizmī also wrote a basic book about algebra called ”al-jabr w’all-muqābala,” which technically meant “bone-setting,” but more generally “restoration and compensation.” When this book was translated into Latin “al-jabr” became “algebra," the word we still use today.

More on Arabic Algebra

Al-Khwārizmī also wrote a basic book about algebra called ”al-jabr w’all-muqābala,” which technically meant “bone-setting,” but more generally “restoration and compensation.” When this book was translated into Latin “al-jabr” became “algebra," the word we still use today.

The most famous of the Arabic mathematicians was the Persian Omar Khayyam, who worked on cubic equations. But his fame rested primarily on the fact that he was a talented poet. His poems much later appeared in English in a famous translation by Sir Richard Burton.

Mathematics Moves West

The Italian Renaissance included a burst of mathematical activity as Greek and Arabic mathematics became known. The Arabic mathematicians knew how to solve linear and quadratic equations, but made limited progress on solving cubic (degree 3) equations. The crucial breakthrough was made in Italy in a series of discoveries and an accompanying soap opera of events.

Mathematics Moves West

The Italian Renaissance included a burst of mathematical activity as Greek and Arabic mathematics became known. The Arabic mathematicians knew how to solve linear and quadratic equations, but made limited progress on solving cubic (degree 3) equations. The crucial breakthrough was made in Italy in a series of discoveries and an accompanying soap opera of events.

(1) First Scipione del Ferro (1465-1526) and then Tartaglia (1500-1557) discovered a method of solving certain classes of cubic equations, but kept it secret to win public competitions.

Act II

(2) Girolamo Cardano, a physician by trade, convinced Tartaglia to share the secret of the cubic with him, promising never to reveal it. But once he knew Tartaglia's method ...

Act II

(2) Girolamo Cardano, a physician by trade, convinced Tartaglia to share the secret of the cubic with him, promising never to reveal it. But once he knew Tartaglia's method ...

(3) Cardano was able to generalize it to a method of solving *any* cubic equation. Feeling it was now mainly his own work, he no longer felt bound to secrecy, and wrote a book, called *Ars Magna* ("The Great Art"), in which he told all. Tartaglia was furious!!

Act II

(2) Girolamo Cardano, a physician by trade, convinced Tartaglia to share the secret of the cubic with him, promising never to reveal it. But once he knew Tartaglia's method ...

(3) Cardano was able to generalize it to a method of solving *any* cubic equation. Feeling it was now mainly his own work, he no longer felt bound to secrecy, and wrote a book, called *Ars Magna* ("The Great Art"), in which he told all. Tartaglia was furious!!

But there was a "little" problem.

Act III

Sometimes Cardano's methods involved square roots of negative numbers (imaginary or complex numbers), even when the answer was real.

Act III

Sometimes Cardano's methods involved square roots of negative numbers (imaginary or complex numbers), even when the answer was real.

(4)The missing step was provided by Rafael Bombelli (1526-1572), who developed a method of dealing with such expressions that we might call the beginning of a theory of complex numbers, numbers of the form $a + bi$.

Act III

Sometimes Cardano's methods involved square roots of negative numbers (imaginary or complex numbers), even when the answer was real.

(4)The missing step was provided by Rafael Bombelli (1526-1572), who developed a method of dealing with such expressions that we might call the beginning of a theory of complex numbers, numbers of the form $a + bi$.

(5) Lodovico Ferrari (1522-1565), a student of Cardano's, showed how to extend Cardano's techniques to solve all equations of degree 4.

Algebra Comes to France

Major progress took place in algebra in the first half of the 17th century, primarily at the hands of two talented Frenchmen, René Descartes (1596-1650), a philospher, and Pierre de Fermat (1601-1665), a lawyer, judge, and amateur mathematician.

Algebra Comes to France

Major progress took place in algebra in the first half of the 17th century, primarily at the hands of two talented Frenchmen, René Descartes (1596-1650), a philospher, and Pierre de Fermat (1601-1665), a lawyer, judge, and amateur mathematician.

(1) Algebraic notation ($+, -, \times, =, x^2$) was invented and solidified into something close to ours. Descartes suggested using letters near the end of the alphabet for unknowns or variables and those near the front for constants.

Algebra Comes to France

Major progress took place in algebra in the first half of the 17th century, primarily at the hands of two talented Frenchmen, René Descartes (1596-1650), a philospher, and Pierre de Fermat (1601-1665), a lawyer, judge, and amateur mathematician.

(1) Algebraic notation ($+, -, \times, =, x^2$) was invented and solidified into something close to ours. Descartes suggested using letters near the end of the alphabet for unknowns or variables and those near the front for constants.

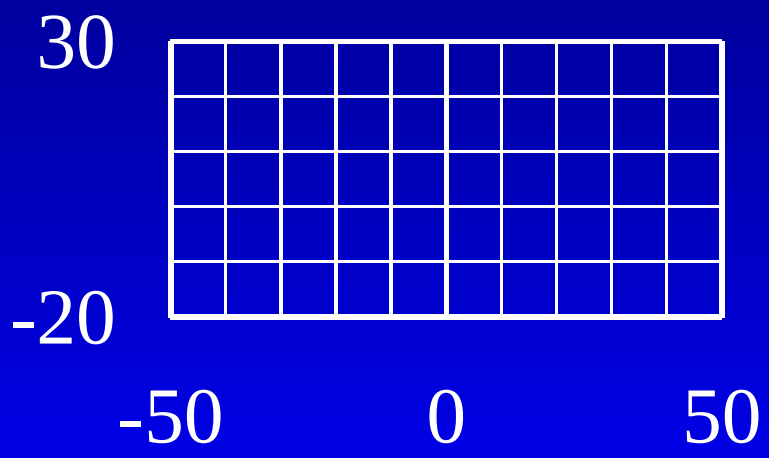
(2) No one could figure out how to solve fifth degree equations, but a general theory of polynomials and their roots evolved.

Analytic Geometry

(3) Fermat and Descartes linked algebra and geometry using what we now call “coordinate geometry” or “analytic geometry” (thus we call the coordinate plane the “cartesian plane).” Both Fermat and Descartes used it to study curves by their equations and showed the power of algebra to solve difficult geometric problems.

Analytic Geometry

(3) Fermat and Descartes linked algebra and geometry using what we now call “coordinate geometry” or “analytic geometry” (thus we call the coordinate plane the “cartesian plane).” Both Fermat and Descartes used it to study curves by their equations and showed the power of algebra to solve difficult geometric problems.



Fermat's Last Theorem

(4) Fermat, after reading the works of Diophantus, introduced a whole new class of algebraic problems in number theory. His most famous "theorem" (unproved) was that

$$x^n + y^n \neq z^n$$

for all positive integers x, y, z and $n \geq 3$.

Fermat's Last Theorem

(4) Fermat, after reading the works of Diophantus, introduced a whole new class of algebraic problems in number theory. His most famous "theorem" (unproved) was that

$$x^n + y^n \neq z^n$$

for all positive integers x, y, z and $n \geq 3$.

This assertion was not proved true until 1994 by Andrew Wiles of Princeton.

Modern Mathematics

Today we have only had time to sketch the history of our math up to the time of the rise of modern mathematics (1600-2004). With the discovery of the calculus by Newton and Leibnitz in the 2nd half of the 17th century, a deep and wide mathematical stream has developed that has resulted one of the greatest and most profound human intellectual achievements and is today practiced worldwide.

Modern Mathematics

Today we have only had time to sketch the history of our math up to the time of the rise of modern mathematics (1600-2004). With the discovery of the calculus by Newton and Leibnitz in the 2nd half of the 17th century, a deep and wide mathematical stream has developed that has resulted one of the greatest and most profound human intellectual achievements and is today practiced worldwide.

Suggested Reading: *A Parrot's Theorem* by Denis Guedj. A novel about a French family who find a parrot and receive a shipment of books that plunges them into both a mystery and a search into "the fascinating story of mathematics."