EXPLORATION
The Origins of Kepler's Third Law
A set of historically-based exercises in basic algebra

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Both observations and theoretical considerations influenced Kepler's earliest attempts to find a mathematical relationship between the radii and the periods of the planetary orbits. In these notes, we examine the data that Kepler used when he composed his first astronomical work, compare it with modern data and consider how Kepler processed and displayed his data in trying to make sense of it. These notes are intended to provide an opportunity to think deeply about the role of mathematics in science by reliving Kepler's conceptual journry from observation to physical law.

Classroom pilots indicate that this is *not yet ready for prime time*. I am looking for people willing to help make this into useable classroom material.

1. Introduction.

Johannes Kepler was born December 27, 1571 in Weil der Stadt, which lies west of Stuttgart in southwestern Germany. In 1589, he entered the University of Tübingen (to the south of Stuttgart). There, his teacher, Michael Maestlin, introduced him to the astronomical theories of Copernicus. After taking his M.A. in 1591, he began studying to become a pastor of the Lutheran Church. In 1594, shortly before completing his studies, he was recommended for a post as mathematics teacher at the Protestant School in Graz, and he accepted.

Kepler's first book on the solar system, *Mysterium cosmographicum* ("The Secret of the Universe") was published in 1596. Its chief aim, as he stated in the preface, was to present a theory tying the number, the positions and the motions of the planets—Mercury, Venus, Earth, Mars, Jupiter and Saturn were the only known planets at the time—to the five Platonic solids. Kepler always remained attached to his theory of the Platonic solids, but in the opinion of of modern astronomers, it is of little significance beyond an historical curiosity. The *Mysterium cosmographicum* advanced other ideas that were far more important. It was the earliest astronomical work to attempt to present a mathematical account of the motions of the planets in terms of physical effects produced by the Sun. Moreover, it opened up lines of thought that would mature and bear fruit in Kepler's later investigations—particularly the question of the relationship of the distances of the planets from the Sun to their orbital periods.

Kepler sent a copy of his book to the Danish astronomer Tycho Brahe, famous for making planetary observations far more accurate than any that had been taken before. Brahe urged a meeting, which took place—after many delays—in Prague, on the first day of the year 1600. At the time, Brahe was trying to understand the orbit of Mars. When Kepler joined him, he bet Brahe that he could solve the problem in a week. It actually took much longer. In 1601, Brahe died. Kepler did not finish his work on Mars until 1605. His findings were published in 1609 in his most important work, *Astronomia nova* ("The New Astronomy").

In terms of impact on the future development of theoretical astronomy, Kepler's greatest accomplishment was his discovery of three mathematical laws that describe the motions of the planets. The first two were expounded in *Astronomia nova*. What today we call "Kepler's First Law" states that each planet follows an elliptic orbit with the Sun at one focus. "Kepler's Second Law" states that as a planet moves along its orbit, the line drawn from the planet to the Sun sweeps out equal areas in equal times. "Kepler's Third Law" was not discovered until nine years later. It states that the cube of the mean distance from a planet to the Sun is proportional to the square of the orbital period of the planet (that is, the time it takes the planet to complete one full orbit). Of it, he writes,

> "... it was conceived on March 8 of the year 1618, but unfelicitously submitted to calculation and rejected as false, and recalled only on May 15, when by a new onset it overcame by storm the darkness of my mind with such full agreement between this idea and my labor of seventeen years on Brahe's observations that at first I believed I was dreaming and had presupposed my result in the first assumptions."[1]

Kepler was one of the most open and unabashed of mathematical writers. He never hesitated to describe blunders, wrong turns and dead ends alongside his true discoveries. We can learn from him what using math in real life to solve real problems is really like. Mathematicians do not always know beforehand how a given problem should be approached. Their work is not always quick, precise and free of error. A spirit of adventure is needed, willingness to throw out ideas and set them down on paper, a thirst for truth and a critical eye that can picks from many ideas the ones that are right. Perhaps, more than anything else, it is the ability to hang on to a problem and keep working until it is solved.[2]

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[1] *Harmonice mundi* (1618) in *Johannes Kepler Gesammelte Werke*, vol. 6 (Munich, 1940), p. 302. Translated in Owen Gingrich, "The origins of Kepler's Third Law," in *Vistas in Astronomy*, ed. by A. Beer and P. Beer, vol. 18, p. 595.

[2] If I may add a personal thought, I am reminded of the Bible story of Jacob wrestling the angel. In this, there is metaphor for the lonely struggle any of us may face when challenged by something beyond our easy understanding. Recognizing the heavenly nature of his adversary, Jacob refused to let go until the angel consented to bless him. Similarly, the true scholar and the true student will wrestle with ideas even if they seem threatening and impossible to master, trusting that the light and grace of understanding will come if only one takes hold and refuses to let go. Kepler was a true scholar, and a perfect example for all of us who accept the challenge of understanding that which is beyond our present

Kepler's scientific career lasted until his death in 1630. His three laws were but a tiny part of his life's work, the aim of which, more than anything else, was to understand the Divine plan of the universe. In the same work where he first expounded his third law, Kepler wrote:

> "If I have been allured into rashness by the wonderful beauty of Thy works, or if I have loved my own glory among men, while I am advancing in the work destined for Thy glory, be gentle and merciful and pardon me: and finally deign graciously to effect that these demonstrations give way to Thy glory and the salvation of souls and nowhere be an obstacle to that."[3]

horizons.

[3] *Harmonice mundi* (1618), *ibid.*, p. 363. Translation by Charles Glen Wallis, in *Great Books of the Western World*, vol. 16 (Chicago, 1952), p. 1080.

2. How far are the planets from the Sun: Copernicus' data *vs.* modern data.

In the *Mysterium cosmographicum*, Kepler accepted the Copernican view of the solar system. He assumed that the planets moved around the sun in orbits that were essentially circular. Much of the data on which the *Mysterium* was based was drawn from Copernicus.

Instead of measuring the orbital radii in absolute terms, Kepler often used the ratios between the radii of the orbits of neighboring planets. He presented the following figures, attributing them to Copernicus:

Saturn:Jupiter ::	1000:572
Jupiter:Mars ::	1000:290
Mars:Earth ::	1000:658
Earth:Venus ::	1000:719
Venus:Mercury ::	1000:500

Table 1: Ratios of orbital radii, as reported by Kepler, 1596.

The table shows that according to Kepler's report of Copernicus' data, if the orbit of Saturn were taken to be 1000 units in radius, then the orbit of Jupiter would be 572 units in radius. If new units were chosen so that Jupiter's orbit were 1000 units in radius, then the orbit of Mars would be 290 units in radius, and so on.

Table 2, below, gives modern measurements of the mean orbital radii (*i.e.*, the average of the closest and furthest distances of the planet from the center of the Sun).[4] Distances are measured in "astronomical units." One astronomical unit is a distance equal to the Earth's mean distance from the Sun. This table shows that Pluto is nearly 40 times as far from the Sun as Earth.

Pluto	39.529402
Neptune	30.061069
Uranus	19.191391
Saturn	9.538762
Jupiter	5.202833
Mars	1.523688
Earth	1.000000
Venus	0.723332
Mercury	0.387099

Table 2: Modern measurements of orbital radii, in astronomical units.

1. *If Table 1 had been prepared from modern data, what would its entries be? If Table 2 were prepared from Kepler's data, what would its entries be?*
2. *Assuming the modern figures are accurate, how accurate were the figures that Kepler was working with? What is the size of the errors in Kepler's data in astronomical units? How large are these errors in comparison to the size of the data? Express the errors as percentages.*

[4] The data comes from: Kenneth R. Lang, *Astrophysical Data: Planets and Stars*, Springer-Verlag, 1992, page 41.

3. How long does it take the planets to go around the Sun?

Chapter XX of the *Mysterium* is titled *Quae sit proportio motuum ad orbes* ("What the ratio of the motions to the orbits is"). It concerns the relative sizes of the planetary orbits and the times required by the planets to complete their orbits. We find there the following table:

	Saturnus	Iupiter	Mars	terra	Venus	Mercurius
	Dies scr.	*Dies scr.*	*Dies scr.*	*Dies scr.*	*Dies scr.*	*Dies scr.*
Saturnus	10759 12					
Iupiter	6l59	4332 37				
Mars	1785	1282	686 59			
terra	1174	843	452	365 15		
Venus	844	606	325	262 30	224 42	
Mercurius	434	312	167	135	115	87 58

Table 3. Reproduced from *Mysterium cosmographicum*.

I wrote the labels in Latin in order to imitate the original as closely as possible. The planet names are clear. *Dies* means days (on earth) and *scr.* is an abbreviation for "scrupula," sixtieths of a day. The number at the top of each column is the time—in days and scrupula—that it takes the planet named above to complete one full circuit about the Sun, according to Kepler's data. For terra, the entry is (as you surely know) 365 and 15/60ths. The table indicates that it takes Saturn $10759\frac{12}{60}$ earth days to complete one orbit about the Sun.

The other entries show how many days the inner planet would require to complete one orbit if it were moving at the speed of the outer. These numbers can be calculated from the ratios in Table 1. For example, multiplying Mars' period by the ratio of Earth's orbital radius to Mars' gives the length of time Earth would require to circle the Sun if it were travelling at the same speed as Mars:

$$686\frac{59}{60} \times \frac{658}{1000} = 452.035$$

This is the second entry in the Mars column. If we multiply this number by the ratio of Earth's orbital radius to Venus', we get 325.013—the number of days Venus would require to circle the Sun if it travelled at the speed of Mars. This is the entry opposite Venus in the Mars column.

1. *There are small differences between the actual entries in Table 3 and the figures one obtains by starting with the data in Table 1 and the column heads in Table 3. Find them. Speculate regarding how they came about.*
2. *Modern measurements of the orbital periods are given in Table 4*[5]*, below. Compare Kepler's data with the modern data. If Table 3 were computed with modern data, what would its entries be?*

[5] From Kenneth R. Lang, *op. cit.*

Pluto	90465
Neptune	60189
Uranus	30685.4
Saturn	10759.22
Jupiter	4332.589
Mars	686.980
Earth	365.256
Venus	224.701
Mercury	87.969

Table 4: Modern measurements of orbital periods, in days.

3. *What does Table 3 show about the speed of a planet further from the Sun? Is it travelling faster or slower than a closer planet? Can you spot a systematic relationship?*
4. *Immediately after presenting Table 3, Kepler computed the ratios of successive orbital periods. His work is shown in the following table*:

Saturn:Jupiter	::	1000:403
Jupiter:Mars	::	1000:159
Mars:Earth	::	1000:532
Earth:Venus	::	1000:615
Venus:Mercury	::	1000:392

Table 5: Ratios of orbital periods,
as computed by Kepler in 1596, from data in Table 3.

Compare these ratios with the ratios in Table 1. If all the planets moved at the same speed, how would Tables 1 and 5 compare? Explain why the ratios in Table 5 are always larger than the ratios in Table 1. What does Table 5 confirm?

5. *Using modern data, make a table showing the speeds the planets travel. You may assume the orbits are circular, but recognize that your results will be only approximate.*

4. Is there a rule that determines a planet's speed?

Kepler sought a reason why the planets further from the Sun were slower. In the first edition of the *Mysterium*, he speculated that the Sun must be the source of the planets' motion, just as it was the source of light. He attributed to the Sun a *motricem animam* ("moving soul"), whose influence weakened at greater distances, just as strength of sunlight grows lesser at greater distances.

The idea of "moving souls" is very interesting historically. The Latin word, *animam* is the same that is used in Latin translations of Aristotle's treatise on plant and animal life, *De Anima*. As Aristotle used the word, the soul is that which imparts life and enables a living thing to do what it properly does—a plant to grow, a dog to run and bark, a person to talk and think—and explore algebra! In Aristotelian and Scholastic cosmology, the heavenly bodies also had souls, which produced and guided their motion. Kepler, reasoning from the data on which he based the *Mysterium*, explicitly rejected the idea that each planet has its *own* soul, and suggested instead that in the solar system there is but a single soul, and it is in the Sun. Here is a break from the ancient cosmological tradition and a first tentative step toward the modern picture, where it is indeed the Sun—by its gravitation—that controls the motions of the planets. In footnotes added to the second edition of the *Mysterium* (which was published in 1621), Kepler shows that he has broken yet further away from the ancient tradition, having discarded the idea of souls altogether. Instead, he supposed that some force—which, like light, is "corporeal" but "immaterial"—must emanate from the Sun and drive the planets.

Whatever the physical reason why the planets slowed further from the Sun, Kepler also wanted to know the mathematical relation between the speeds. Since each planet's speed determines its period (and vice versa), this is no different from the problem of finding a mathematical relation between the periods. Can one compute the orbital periods from the radii?

In the *Mysterium*, Kepler argued as follows. When we go from a chosen planet to the next planet further from the Sun, two things contribute to the increase of the period. First, the planet must travel a greater distance; second, the influence of the motive power of the Sun decreases, causing the speed to lessen. He concludes,

> "Hence it follows that one excess in the distance of a planet from the Sun acts twice over in increasing the period: (6) and conversely, the increase in the period is double the difference in the distances."[6]

The "(6)" in this passage refers to a footnote added in the second edition. We shall return to the footnote in the next section, but first let us understand what he is saying here.

By "increase" and "difference," he means proportionate increase and difference. If the inner planet has orbital radius r_0 and period T_0 and the outer planet has orbital radius r_1 and period T_1, the proportionate increase in the period is $\frac{T_1 - T_0}{T_0}$ and the proportionate

[6] Johannes Kepler, *Mysterium Cosmographicum*, translated by A. M. Duncan, Abaris Books, New York, 1981, p. 201. (This book is a translation in parallel with a photo-reproduction of the 1621 edition; the passage whose translation is quoted appears on page 76 of the 1621 edition.

difference in the radius is $\frac{r_1 - r_0}{r_0}$. Kepler is saying that

$$\frac{T_1 - T_0}{T_0} = 2\frac{r_1 - r_0}{r_0}. \tag{1}$$

In the next paragraph, Kepler writes:

> "Therefore, adding half the increase to the smaller period should show the true ratio of the distances: the sum is proportional to the distance of the superior planet, and the simple lesser period represents the distance of the inferior, that is, of its own planet, in the same proportion. For example: the periodic motion of Mercury takes about 88 days, and that of Venus about $224\frac{2}{3}$ days. The difference is $136\frac{2}{3}$ days and half that is $68\frac{1}{3}$. Adding that to 88 makes $156\frac{1}{3}$. Then as 88 is to $156\frac{1}{3}$, so the radius of the mean circle of Mercury is to the mean distance of Venus."[7]

Here, we see Kepler rephrasing relation (1) in the following form

$$\frac{r_1}{r_0} = \frac{T_0 + (1/2)(T_1 - T_0)}{T_0}. \tag{1$'$}$$

Kepler's intuitive grasp of algebra is evident in these passages, but his physical intuitions seem to have been out of whack. There is no apparent reason why two influences should produce exactly double the effect of one acting alone. What is worse, (1) does not state a universal relation between orbital periods and radii. It only relates the periods and radii of pairs of planets. The following exercises illustrate this.

1. *Confirm that* (1′) *is indeed what Kepler is asserting in the quoted passage. Show that* (1) *and* (1′) *are indeed different formulations of the same relation, and are equivalent to:*

$$\frac{r_1}{r_0} = \frac{(T_1 + T_0)/2}{T_0}. \tag{1$''$}$$

   Note that (1″) *simply says that* r_1 *is to* r_0 *as the average of* T_0 *and* T_1 *is to* T_0*.*
2. *Take the distances from Earth, Venus and Mercury to the Sun to be respectively* 1.0, .723 *and* .387 *astronomical units. Assume Mercury's period is* 88 *days. Show that* (1) *predicts a period of* 240.8 *days for Venus. If we use this figure for the period of Venus, and attempt to predict the Earth's period, show that we get* 425.3*. If, on the other hand, we attempt to compute Earth's period directly from Mercury by* (1)*, we get* 366.8*. Explain why the formula* (1) *yields two different results from the same data.*
3. *Choose one planet as a reference, and call its radius and period* r_0 *and* T_0*. Show that* (1) *yields the following linear law relating the radius* r *and period* T *of an arbitrary planet:*

$$T = \left(\frac{2T_0}{r_0}\right) r - T_0.$$

[7] *ibid.*

Show that if a different "reference planet"—with radius r_1 and period T_1 is chosen—then the resulting linear function,

$$T = \left(\frac{2T_1}{r_1}\right) r - T_1,$$

is different.

4. *Kepler gives the following table of radial ratios, computed from the periods by following the pattern in the quotation following* (1)*:*

Saturn:Jupiter ::	1000:574
Jupiter:Mars ::	1000:274
Mars:Earth ::	1000:694
Earth:Venus ::	1000:762
Venus:Mercury ::	1000:563

Table 6: Ratios of orbital radii as predicted by (1), from the data in Table 3, (Kepler, 1596).

Check these entries using a pocket calculator.

To compute the radius of a planet's orbit from (1), we need not only the planet's period, but also period and radius data for a second planet. And the result we get depends on which second planet we chose. (Kepler required that we choose the next planet toward the Sun.) This makes (1) unsatisfactory as a physical law describing orbital motion in an arbitrary planetary system. It does not treat the planets in a symmetric fashion. Its predictions depend on which planet we choose as a reference.

Physicists know that good physical laws do not introduce artificial distinctions, but respect whatever symmetry is present in the phenomena. Kepler, it must be remembered, viewed the solar system as a unique creation; he even believed that the number of the planets (6, he thought) must have a mathematical explanation. So, in seeking a mathematical order, it was not unreasonable for him to propose a relation in which the planets played unique roles. It is remarkable, therefore, that ultimately he hit upon three laws that apply universally to any system of satellites, *e.g.*, the moons of Jupiter.

5. The arithmetic versus the geometric mean.

Now to the footnote "(6)." Remember that this was inserted just before Kepler introduced the relation (1). Kepler at age 50 comments on Kepler at age 25:

> "Here the mistake begins. For this is not the exact converse of what precedes, that is, that the distance of the Sun makes a double contribution to the increase of the period. Now what I ought to have inferred, together with its converse, is that the ratio of the periods is the square of the ratio of the distances, ... [This] was the legitimate conclusion from this line of argument."[8]

Kepler is saying that (1) should never have been proposed in the first place. His youthful reasoning should have led him to the relation:

$$\frac{T_1}{T_0} = \left(\frac{r_1}{r_0}\right)^2. \tag{2}$$

He is not claiming that this is the correct relationship, but only that this is what the theoretical considerations should have led to.

1. *The footnote quoted above continues: "... You see how at this point the arithmetic mean was taken, ..., when the geometric mean should have been taken." The arithmetic mean of T_0 and T_1 is $(T_0 + T_1)/2$; the geometric mean is $\sqrt{T_0 T_1}$. Show that relation (2) results when relation (1″) is altered this way.*

2. *Can the geometric mean of two positive numbers ever be equal to the arithmetic mean? Can the geometric mean of two positive numbers be smaller than the arithmetic mean? Can it be larger?*

3. *In the next footnote, Kepler points out that even though (2) is the relation suggested by the theory, relations (1)–(1″) fit the data better. Is this true? What would Table 6 look like if it were computed using Table 3 and (2) instead of (1″)?*

[8] *op.cit.*, pp. 204-5.

6. Power laws.

A *power law* is a function p of a positive variable x of the form

$$p(x) = c\,x^k,$$

where c and k are constants. The area of a circle is given by a power law:

$$a = \pi\, r^2,$$

where a denotes the area and r the radius. So is the volume v of a sphere of radius r:

$$v = \frac{4}{3}\,\pi\, r^3,$$

and the surface area s of a sphere of radius r:

$$s = 4\,\pi\, r^2.$$

Another example is the formula that gives the volume of the sphere as a function of its surface area:

$$v = \frac{1}{6\sqrt{\pi}}\, s^{3/2}.$$

Any power law gives rise to a relation between ratios that is similar to equation (2) of section 5. Suppose $p(x) = c\,x^k$. Then for any positive arguments x_0 and x_1

$$\frac{p(x_1)}{p(x_0)} = \left(\frac{x_1}{x_0}\right)^k.$$

Conversely, if f is a function such that for any positive arguments x_0 and x,

$$\frac{f(x)}{f(x_0)} = \left(\frac{x}{x_0}\right)^k,$$

then

$$f(x) = \frac{f(x_0)}{x_0^k}\, x^k,$$

so f is a power law. Thus, a function is a power law if and only if the ratio of any two values is a power of the ratio of the corresponding arguments.

1. *Areas of similar triangles are related to their linear dimensions by a power law. Pick a particular triangle $\triangle QRS$. Let b_0 denote the length of QR, and let h_0 denote the length of the altitude from S. Then, the area A_0 is given by $A_0 = \frac{1}{2}\, b_0\, h_0$. Now suppose that b and h are the lengths of the corresponding edge and altitude of a triangle similar to $\triangle QRS$. Then*

$$\frac{h}{b} = \frac{h_0}{b_0},$$

so

$$h = \frac{h_0}{b_0}\, b.$$

This enables us to express the area A of any triangle similar to $\triangle QRS$ as a constant times a power of the length b of its base. Provide the details.

2. *Write the power law that gives the volume of a cube as a function of its surface area.*

7. More on power laws.

This section is more technical than the rest of the exploration, requiring some calculus. You can complete the exploration without it, since nothing further depends on it.

Kepler began his search for a mathematical relation between distances and periods by looking for a rule that relates ratios of distances to ratios of periods. We are going to show that by setting out this way, he was bound to find a power law if he found anything at all. Was it luck that sent him on the correct path, or some deep understanding of the nature of physical law that he did not have the language to express? It is a true mystery.

We say a function f from the positive reals to the positive reals is *multiplicative* if for all positive reals x and y,

$$f(x\,y) = f(x)\,f(y).$$

For example, $f(x) = x^k$ is multiplicative, since $(x\,y)^k = x^k\,y^k$. Not every power law $g(x) = c\,x^k$ is multiplicative—only those with $c = 1$.

Theorem: If f is a continuous multiplicative function from the positive reals to the positive reals, then $f(x) = x^{\ln(f(e))}$, where e is Euler's constant, and ln denotes the natural logarithm. Thus, any continuous multiplicative function from the positive reals to the positive reals is a power law with coefficient 1.

Proof: Suppose $f : \mathbf{R}^+ \to \mathbf{R}^+$ is multiplicative. One checks easily that $f(x^n) = (f(x))^n$ for any integer n. Using the fact that positive qth roots in $\mathbf{R}$ exist and are unique, one can show that $f(x^{p/q}) = (f(x))^{p/q}$ for any integer p and any integer $q \neq 0$. Now

$$\begin{aligned}
f(x) &= f(e^{\ln(x)}) \\
&= f(\lim_{p/q \to \ln(x)} e^{p/q}) \\
&= \lim_{p/q \to \ln(x)} f(e^{p/q}) \\
&= \lim_{p/q \to \ln(x)} f(e)^{p/q} \\
&= f(e)^{\ln(x)} \\
&= x^{\ln(f(e))}.
\end{aligned}$$

Q.E.D.

Let g be a function from the positive reals to the positive reals. We say that g *obeys a ratio rule* if there is a function F such that for any two positive reals x and y,

$$\frac{g(x)}{g(y)} = F\Big(\frac{x}{y}\Big).$$

As we noted in the last section, any power law $g(x) = c\,x^k$ obeys the ratio rule $\frac{g(x)}{g(y)} = (\frac{x}{y})^k$, and conversely a function which obeys a ratio rule *in which F is a simple power (i.e., $F(r) = r^k$)* is a power law with the same exponent as F.

What if we do not know that F is a power function? It turns out that if g is continuous then we don't need to know anything at all about F to be able to conclude that g is a power law.

Theorem: Suppose that g is a continuous function from the positive reals to the positive reals and suppose there is a function F such that for any two positive reals x and y,

$$\frac{g(x)}{g(y)} = F\Big(\frac{x}{y}\Big).$$

Then $g(x) = g(1)\, x^{\ln(g(e)/g(1))}$. In other words, any continuous function from the positive reals to the positive reals that obeys a ratio rule is a power law.

Proof: Let $h(x) = g(x)/g(1)$. Then for any positive reals x and y,

$$\frac{h(x\,y)}{h(y)} = F\Big(\frac{x\,y}{y}\Big) = F\Big(\frac{x}{1}\Big) = h(x),$$

so

$$h(x\,y) = h(x)\,h(y).$$

Hence, h is multiplicative, and since h is clearly continuous, $h(x) = x^{\ln(h(e))}$. Q.E.D.

To summarize, suppose that y depends continuously on x. If we can predict the ratio $\frac{y_1}{y_0}$ from the ratio $\frac{x_1}{x_0}$ *by any rule whatever*, then y is related to x by a power law.

8. Kepler's Third Law.

By 1618, when he discovered the third law, the data available to Kepler from Tycho Brahe's observations was more accurate than the Copernican data Kepler used when writing the *Mysterium*. Below are the mean orbital radii and orbital periods, as reported by Kepler in his *Harmonice Mundi*, which was published in 1619. It is interesting to compare the table below with Tables 1 and 2, to see how accurate Brahe was, and how greatly he improved on the previous figures.

	Mean Orbital Radius (in AU)	Period (in days)	Period (in years)
Saturn	9.510	$10759\ \frac{12}{60}$	29.4571
Jupiter	5.200	$4332\ \frac{37}{60}$	11.8621
Mars	1.524	$686\ \frac{59}{60}$	1.8809
Earth	1.000	$365\ \frac{15}{60}$	1.0000
Venus	0.724	$224\ \frac{42}{60}$	0.6152
Mercury	0.388	$87\ \frac{58}{60}$	0.2408

Table 7: Mean orbital radii (in astronomical units) and orbital periods (in Earth days and Earth years), as reported by Kepler 1619.

1. *There is nothing left for you to do but to read the law from the data. I won't tell you any more about the law than I have already. I leave it to you to discover it, as Kepler himself did, by pouring over the data, and I will let you state it precisely.*

> "...the occasions by which people come to understand celestial things seem to me not much less marvellous than the nature of the celestial things itself."[9]

Acknowledgement. This exposition is much indebted to Owen Gingrich, "The origins of Kepler's Third Law," in *Vistas in Astronomy*, ed. by A. Beer and P. Beer, vol. 18, 1971. This is reprinted in Owen Gingrich, *The Eye of Heaven*, American Institute of Physics, 1993.

Suggested readings. Arthur Koestler's book *The Sleepwalkers* contains vivid biographies of Kepler, Brahe and other figures of the Scientific Revolution. Two of Kepler's astronomical works exist in English translation, and they are fascinating reading. They are referenced in footnotes 3 and 9.

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[9] Johannes Kepler, *New Astronomy*, translated by William H. Donahue, Cambridge University Press, 1992, p. 95.