Calculus 1551, Section 4. August 24, 2009

Notes

The integers: $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$.

The rational numbers: $\mathbb{Q} = \{\, a/b \mid a, b \in \mathbb{Z}, b \neq 0 \,\}$.

The real numbers: $\mathbb{R}$ = "all the numbers on the number line." *But how do you locate a number on the number line?*

Putting integers on a line

1) Pick a line ℓ.
2) Pick a segment U as a unit of measure.
3) Pick a point O on ℓ and a direction on ℓ.
4) Using U mark the integers on ℓ, placing 0 and O and increasing in the chosen direction.

Locating the rational numbers on the line

If we want to find the location of a particular rational number, say $15/7$, we divide U into 7 equal pieces, and we find the point we reach by putting 15 copies end-to end.

How about decimal notation?

You certainly are aware that in decimal notation, some numbers have infinite names. For example:

- $\pi = 3.1415926\ldots$
- $\frac{1}{3} = 0.333\ldots$
- $2.142857142857142857\ldots$ (What is this number? You've seen it before!)
- $0.999\ldots$ (What is this number?)
- $0.101001000100001000001\ldots$ (Is this number rational or irrational?)

When we write a non-teminating decimal, we get some information about the location of the number from each truncation. When we say "between", below, we mean the inclusive between.

- $\pi = 3.???\ldots \Leftrightarrow \pi$ is between 3 and 4
- $\pi = 3.1???\ldots \Leftrightarrow \pi$ is between $3 + \frac{1}{10}$ and $3 + \frac{2}{10}$
- $\pi = 3.14???\ldots \Leftrightarrow \pi$ is between $3 + \frac{1}{10} + \frac{4}{100}$ and $3 + + \frac{1}{10} + \frac{5}{100}$

What number is:

- between 0 and 1
- between $\frac{3}{10}$ and $\frac{4}{10}$
- between $\frac{3}{10} + \frac{3}{100}$ and $\frac{3}{10} + \frac{4}{100}$
- between $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$ and $\frac{3}{10} + \frac{3}{100} + \frac{4}{1000}$
- *etc.*

What number is:

- between 0 and 1
- between $\frac{9}{10}$ and 1
- between $\frac{9}{10} + \frac{9}{100}$ and 1
- between $\frac{9}{10} + \frac{9}{100} + \frac{9}{1000}$ and 1
- *etc.*