



A. The picture above shows a portion of the unit circle in the x - y -plane, as well as a ray OB that makes a positive angle of θ radians with the positive x -axis.

- 1) What are the lengths of the following segments: OB , OD , BD , DA , AC ?
- 2) What are the coordinates of the following points: B , C , D ?
- 3) What is the area of triangle OAB ?
- 4) What is the area of sector OAB ? (Hint: This sector makes up a fraction of the circle equal to $\frac{\theta}{2\pi}$, while the area of the circle is π .)
- 5) By comparing the areas of triangle OAB and sector OAB , show that $0 < \sin \theta < \theta$ when θ is a small positive angle.
- 6) READ the Squeeze Theorem on page 99. What does 5) imply about $\lim_{\theta \rightarrow 0^+} \sin \theta$? Is the sine function continuous from the right at 0?
- 7) Is it continuous at 0?
- 8) By comparing the length of segment DA with the length of segment DB , show that $1 - \theta < \cos \theta < 1$ for small positive θ .
- 9) What does this imply about $\lim_{\theta \rightarrow 0^+} \cos \theta$?
- 10) Is the cosine function continuous at 0?

Morals: a) You got some experience with a representation of the sine and cosine in the unit circle. b) You saw that sine and cosine are both continuous at 0. c) You learned to use the Squeeze Theorem.