M1551 **Test 1.** September 4, 2009 Name: Answers.

Instructions. Do problems in space provided (continuing on back if necessary). This is a 100-point test.

1. [5 points] Using the absolute-value symbol and < (or >), express the following statement in mathematical symbols: "the distance from W to 3 is strictly greater than 5".

$$|W-3| > 5$$

2. [10 points] Describe as a union of intervals: the set of all x such that $|2x-9| \geq 100$.

$$|2X-9| \geq 100 \Leftrightarrow |X - \tfrac{9}{2}| \geq 50$$

$$\Leftrightarrow \begin{cases} X \geq 50 + 9/2 \\ \text{or } X \leq \frac{9}{2} - 50 \end{cases}$$

$$\Leftrightarrow X \in (\infty, 45\tfrac{1}{2}] \cup [54\tfrac{1}{2}, \infty)$$

$$|2X-9| \geq 100 \Leftrightarrow$$

$$\Leftrightarrow 2X-9 \geq 100 \text{ or } -2X+9 \geq 100$$

$$\Leftrightarrow X \geq \frac{109}{2} \text{ or } X \leq \frac{91}{2}$$

$$\Leftrightarrow X \in [54\tfrac{1}{2}, \infty) \cup (-\infty, 45\tfrac{1}{2}]$$

In problems 3 and 4, let ℓ be the line through (p, p^2) and (q, q^2), where p and q are any real numbers.

3. [5 points] Write the equation for ℓ in slope-intercept form ($y = mx + b$):

$$\text{Slope} = m = \frac{p^2 - q^2}{p - q} = p + q.$$

$$\text{Eq. of } \ell \text{ is: } y - p^2 = (p+q)(x-p)$$

$$\text{or: } y = (p+q)x - p(p+q) + p^2$$

$$\text{or: } \boxed{y = (p+q)x + (-pq)} \ (*)$$

(here, $b = -pq$).

4. [10 points] Express as a function of p: the value that q must have in order for ℓ to pass through $(0, 1)$:

If ℓ passes through $(0,1)$, then

$$1 = (p+q)(0) + (-pq) \quad \text{by } (*).$$

i.e., $1 = -pq$.

Thus, if ℓ passes through $(0,1)$, (p,p^2) and (q,q^2), then $\boxed{q = \frac{-1}{p}}$.

5. [5 points] Write as a mathematical expression: "the average rate of change of $f(x)$ on the interval from $x = 6.9$ to $x = 7$."

$$\boxed{\frac{f(7) - f(6.9)}{7 - 6.9}} = 10\,(f(7) - f(6.9))$$

ok

6. [5 points] Express as a limit: "the instantaneous rate of change of $f(x)$ at $x = 7$."

$$\lim_{t \to t} \frac{f(t) - f(7)}{t-7} \quad \underline{\text{or}} \quad \lim_{h \to 0} \frac{f(7+h) - f(7)}{h}$$

7. [10 points] An object falls from 256 feet, so its height after t seconds of falling is $256 - 16t^2$ feet. At what time does it hit the ground? Express as a limit: the object's instantaneous velocity when it hits the ~~ground.~~ Evaluate this limit.

Hits ground when $0 = 256 - 16t^2$, i.e., when $\boxed{t = 4}$. 2

$$\underbrace{\lim_{t \to 4} \frac{(256 - 16t^2) - 0}{t - 4}}_{4} = \lim_{t \to 4} \frac{16(16 - t^2)}{t-4} = \lim_{t \to 4} \frac{16(4-t)(4+t)}{t-4}$$

$$= \lim_{t \to 4} -16(4+t) = -128 \quad 4$$

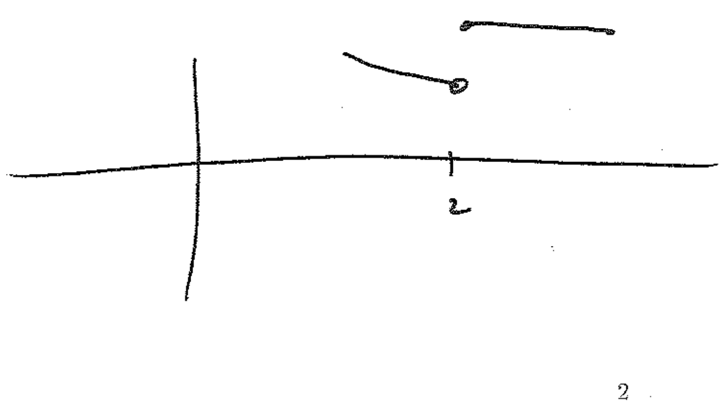
8. [5 points] Explain the meaning of the expression "$\lim_{x \to 3} f(x) = 5$" in plain English, *without* using the word "limit."

f(x) can be made as close as you wish to 5 by restricting ~~x~~ x to a sufficiently small punctured neighborhood of 3

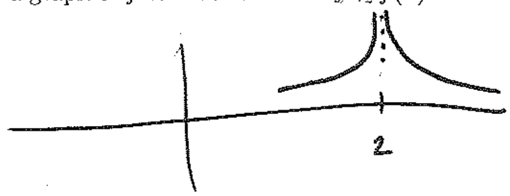
9. [5 points] Explain the meaning of the expression "$\lim_{x \to \infty} f(x) = 8$" in plain English, *without* using the word "limit."

f(x) can be made as close as you wish to 8 by restricting x to an interval of the form $(M, +\infty)$.

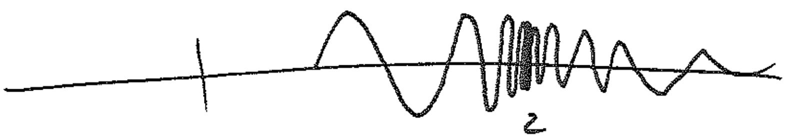
10. [5 points] Draw a graph of f to illustrate: "$\lim_{x \to 2} f(x)$ does not exist because the one-sided limits are different."



11. [5 points] Draw a graph of f to illustrate: "$\lim_{x\to 2} f(x)$ does not exist because f has an asymptote."



12. [5 points] Draw a graph of f to illustrate: "$\lim_{x\to 2} f(x)$ does not exist because f oscillates."



10. Find the limit if it exists. If it does not exist, explain why.

a. [5 points] $\lim_{x\to 3} \frac{x^2-9}{x-3}$ $= \lim_{x\to 3} x+3 = 6$

b. [5 points] $\lim_{x\to 3} \frac{x^2-9}{x-4}$ $= 0$

c. [5 points] $\lim_{x\to 5} \frac{x^2-9}{x-4}$ $= 16$

d. [5 points] $\lim_{x\to 3} \frac{x^2-4}{x-3}$ DNE { denom $\to 0$ & num $\to 5$ as $x\to 3$, so the quantity gets arbitrarily far from 0. }

e. [5 points] $\lim_{x\to 0} \cos\frac{1}{x}$ DNE. oscillates (like $\sin\frac{1}{x}$).