Lemma. For any real numbers a and b, $|a + b| \leq |a| + |b|$.

Proof.

Proposition. Suppose $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_0$ is a polynomial of degree n with leading coefficient 1. Let M be a positive real number. Suppose

$$x > 1 + M + |a_{n-1}| + \cdots + |a_0|. \tag{$*$}$$

Then $f(x) > M$.

Proof. If $(*)$ holds, then ....

Corollary. Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ is a polynomial of degree n with leading coefficient $a_n > 0$. Let M be a positive real number. Then there is x_0 such that: if $x > x_0$, then $f(x) > M$.

Proof.