1. Let $f(x,y) = \frac{x^2}{4} + \frac{y^2}{9}$. Write the equations for the level curve that passes through $(2,3)$ and for the level curve that passes through $(0,6)$. Draw these curves.
2. Let $h(x,y) = \sin(x\,y^2)$. Compute $h_{xx}(0,1)$ and $h_{xy}(0,1)$ and $h_{yy}(0,1)$.
3. Find the equation for the tangent plane to the graph of $z = e^x \sin y$ at $(1, \pi/2, e)$.
4. Suppose $x = r\cos\theta$ and $y = r\sin\theta$ and $z = f(x,y)$. Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ in terms of r, θ, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
5. a. Find the instantaneous rate of change of $z = x + xy$ in the direction of the vector $(3/5, 4/5)$ at the point $(2,5)$.

   b. Find $\frac{\partial z}{\partial u}|_{(0,0)}$, where u has the same direction as $(1,2)$.
6. At the point $(1,2)$, in what direction does the function $f(x,y) = x^3y - xy^2$ increase most rapidly, and what is the rate of increase?
7. Find the equation of the plane tangent to the surface $\frac{\pi}{2} + 1 = x + y - z + \sin(xyz)$ at $(1, \frac{\pi}{2}, 1)$.
8. Do each part of the following:

   a) Let $f(x,y)$ be a differentiable function on the plane. Suppose $f(a,b) = c$. Write the equation for the tangent plane to the surface $z = f(x,y)$ at the point (a,b,c).

   b) Let $F(x,y,z) = f(x,y) - z$. What is the relationship between the level surface of F at 0 and the graph of f?

   c) Compute $\nabla F(a,b,c)$.

   d) Recall from M1552 that if $\mathbf{n}$ and $\mathbf{v}_0$ are vectors in space, then the plane that is normal to $\mathbf{n}$ and passes through $\mathbf{v}_0$ is described by the the equation in vector form $\mathbf{n} \cdot (\mathbf{v} - \mathbf{v}_0) = 0$. Write in vector from the equation for the plane that is normal to $\nabla F(a,b,f(a,b))$ and passes through (a,b,c).

   e) Convert the vector equation in part c) into algebraic form, *i.e.*, let $\mathbf{v} = (x,y,z)$, and write out the vector equation in terms of x, y and z.

   f) Bonus problem. Observe that planes that the answers to a) and e) are the same. Explain why.
9. Find and classify the critical points of

   a. $f(x,y) = x^3 + x^2 - y^2$

   b. $h(x,y) = x^3 - 6xy + 8y^3$
10. Find the locations of the points where $f(x,y,z) = z - x - y$ has maximum and minimum values and find those values, subject to the constraint $g(x,y,z) = x^2 + y^2 + z^2 = 50$.