M2057 Test 2. Sample **Use to review for test on October 22, 2003**

1. Integrate: $\int_0^1 \int_0^{2x} \sqrt{1+x^2}\, dy\, dx$.

2. Let R be the triangle with vertices at $(-1,0)$, $(1,0)$ and $(0,1)$. Express $\int\int_R f(x,y)\, dA$ as an iterated integral in each of the two possible orders of integration. (Note: in one order, it will be written as a sum of two iterated integrals.)

3. Draw the region of integration and change the order of integration:

$$\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx.$$

4. Let D be the disk of radius 1 about $(0,0)$. Evaluate using polar coordinates:

$$\int\int_D \sqrt{1+x^2+y^2}\, dA.$$

5. Find the center of mass of the lamina that occupies the region in the first quadrant between the circle of radius 1 and the circle of radius 2 if the density is $\frac{1}{x^2+y^2}$.

6. Find $\int\int\int_E 2\, y\, z\, dV$ where E is the region of space bounded by the vertical planes $x=0$, $y=0$ and $y=2-2x$, bounded on the bottom by the x-y-plane and bounded on top by the surface $z=\sqrt{x+y}$.

7. Use spherical coordinates to compute $\int\int\int_E z\, dV$, where E is the region of space inside the cone $\phi=\frac{\pi}{4}$ and between the spheres $\rho=1$ and $\rho=2$.

8. Using the fact that the transformation $T(u,v)=(a\,u+b\,v, c\,u+d\,v)$ maps the unit square $S=\{\,(u,v)\mid 0\le u\le 1\ \&\ 0\le v\le 1\,\}$ to the parallelogram R with vertices at $(0,0)$, (a,c), $(a+b,c+d)$, (b,d), use the change of variables formula to convert $\int\int_R x\, y\, dA$ to an integral of the form $\int\int_S H(u,v)\, dA$. (Figure out what H must be.)