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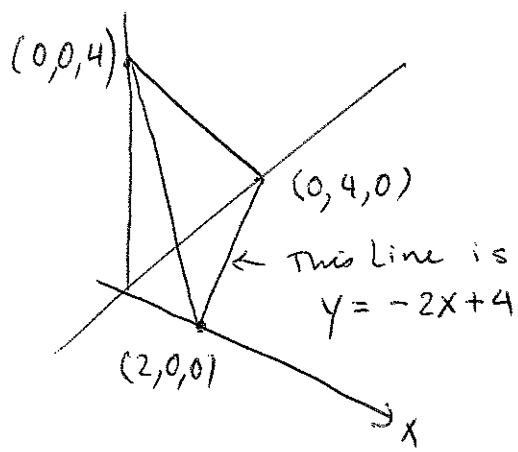
The region is bounded by $x=0$, $y=0$, $z=0$ and $2x+y+z=4$.



We decide to integrate in the order $\iiint dz\,dy\,dx$.

In this region, $0 \le x \le 2$.

For fixed x, y can be as small as 0 and as large as $-2x+4$.

For fixed x and y, z can be as small as 0 and as large as $-2x-y+4$

$$\text{Volume} = \iiint_E 1\, dV .$$

$$= \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 1 \, dz\, dy\, dx$$

$$= \int_0^2 \int_0^{4-2x} 4-2x-y \; dy\, dx = \int_0^2 (4-2x)y - \frac{y^2}{2} \Big|_{y=0}^{4-2x} dx$$

$$= \int_0^2 (4-2x)^2 - \frac{(4-2x)^2}{2} dx \overset{*}{=} \int_0^4 \frac{u^2}{2} \cdot \frac{1}{2} dx$$

$dx = \frac{-1}{2} du$
We will get rid of the minus sign by changing order of integration at (*)

$u = 4-2x$
$du = -2dx$
$x=0 \Rightarrow u=4$
$x=2 \Rightarrow u=0$

$$= \frac{u^3}{3 \cdot 4} \Big|_0^4 = \frac{64}{3 \cdot 4} = \frac{16}{3}$$