M2056 Test 1 09/17/2012 Name_____

1.a. Draw and label the level sets at $k = 1$, $k = 4$ and $k = -4$ of $f(x, y) = (y - x)^2$.

1.b. Sketch the graph of $z = -4x - 3y + 12$ and describe the shape of the graph in a sentence.

2. Compute the partial derivatives:

a) $\frac{\partial}{\partial x}\, x \sin(x\, y) =$

b) $\frac{\partial}{\partial y}\, x \sin(x\, y) =$

c) $\frac{\partial^2}{\partial y\, \partial x}\, (x^5 y^7 + x^2 y^3) =$

3. Find the equation for the plane tangent to $z = \frac{x\,y}{x+y}$ at the point $(6, 3, 2)$.

4. Find $\frac{d}{dt} f(\mathbf{c}(t))$ when $t = \pi/2$ if $f(x, y) = 3x - y^2$ and $\mathbf{c}(t) = (\cos t, t)$.

5. Find the directional derivative of $z = x + x^2y + y^2$ in the direction $u = (3/5, 4/5)$ at the point $(x, y) = (2, 1)$.

$$\frac{\partial z}{\partial u} \Big|_{(2,1)} =$$

6. Suppose $z = f(s,t)$ and $s = g(u,v)$ and $t = h(u,v)$.

a) Express $\frac{\partial z}{\partial u}$ in terms of $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$, $\frac{\partial s}{\partial u}$, $\frac{\partial t}{\partial u}$.

$$\frac{\partial z}{\partial u} =$$

b) Suppose $f_s(3,4) = 5$, $f_t(3,4) = 7$, $g(0,1) = 3$, $h(0,1) = 4$, $g_u(0,1) = 10$, $h_u(0,1) = 20$. What is the numerical value of the following?

$$\left.\frac{\partial z}{\partial u}\right|_{\substack{u=0\\v=1}} =$$

7. Find the critical points of the following functions, and determine whether they are local maxima, minima or saddle points.

(a) $f(x,y) = x^2 + 2xy + 5y^2 + 8x$

(b) $f(x,y) = x^2 + 4xy + 2y^2 - 8x$

8. Using Lagrange Multipliers, find the maximum and minimum values of $z = 3x - 2y$ on the circle $4 = x^2 + y^2$.