4.35. A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you win −\$1.00. (That is, you lose \$1.00.) Calculate
(a) the expected value of the amount you win;
(b) the variance of the amount you win.

4.38. If $E[X] = 1$ and $\mathrm{Var}(X) = 5$, find
(a) $E[(2 + X)^2]$;
(b) $\mathrm{Var}(4 + 3X)$.

4.39. A ball is drawn from an urn containing 3 white and 3 black balls. After the ball is drawn, it is replaced and another ball is drawn. This process goes on indefinitely. What is the probability that, of the first 4 balls drawn, exactly 2 are white?

4.40. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?

4.41. A man claims to have extrasensory perception. As a test, a fair coin is flipped 10 times and the man is asked to predict the outcome in advance. He gets 7 out of 10 correct. What is the probability that he would have done at least this well if he had no ESP?

4.42. Suppose that, in flight, airplane engines will fail with probability $1 - p$, independently from engine to engine. If an airplane needs a majority of its engines operative to complete a successful flight, for what values of p is a 5-engine plane preferable to a 3-engine plane?

4.43. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability .2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be wrong when decoded? What independence assumptions are you making?

4.44. A satellite system consists of n components and functions on any given day if at least k of the n components function on that day. On a rainy day with probability p_1, whereas on a dry day they each independently function with probability p_2. If the probability of rain tomorrow is α, what is the probability that the satellite system will function?

4.45. A student is getting ready to take an important oral examination and is concerned about the possibility of having an "on" day or an "off" day. He figures that if he has an on day, then each of his examiners will pass him, independently of each other, with probability .8, whereas if he has an off day, this probability will be reduced to .4. Suppose that the student will pass the examination if a majority of the examiners pass him. If the student feels that he is twice as likely to have an off day as he is to have an on day, should he request an examination with 3 examiners or with 5 examiners?

4.46. Suppose that it takes at least 9 votes from a 12-member jury to convict a defendant. Suppose also that the probability that a juror votes a guilty person innocent is .2, whereas the probability that the juror votes an innocent person guilty is .1. If each juror acts independently and if 65 percent of the defendants are guilty, find the probability that the jury renders a correct decision. What percentage of defendants is convicted?

4.47. In some military courts, 9 judges are appointed. However, both the prosecution and the defense attorneys are entitled to a peremptory challenge of any judge, in which case that judge is removed from the case and is not replaced. A defendant is declared guilty if the majority of judges cast votes of guilty, and he or she is declared innocent otherwise. Suppose that when the defendant is, in fact, guilty, each judge will (independently) vote guilty with probability .7, whereas when the defendant is, in fact, innocent, this probability drops to .3.
(a) What is the probability that a guilty defendant is declared guilty when there are (i) 9, (ii) 8, and (iii) 7 judges?
(b) Repeat part (a) for an innocent defendant.
(c) If the prosecution attorney does not exercise the right to a peremptory challenge of a judge, and if the defense is limited to at most two such challenges, how many challenges should the defense attorney make if he or she is 60 percent certain that the client is guilty?

4.48. It is known that diskettes produced by a certain company will be defective with probability .01, independently of each other. The company sells the diskettes in packages of size 10 and offers a money-back guarantee that at most 1 of the 10 diskettes in the package will be defective. The guarantee is that the customer can return the entire package of diskettes if he or she finds more than one defective diskette in it. If someone buys 3 packages, what is the probability that he or she will return exactly 1 of them?

4.49. When coin 1 is flipped, it lands on heads with probability .4; when coin 2 is flipped, it lands on heads with probability .7. One of these coins is randomly chosen and flipped 10 times.
(a) What is the probability that the coin lands on heads on exactly 7 of the 10 flips?
(b) Given that the first of these ten flips lands heads, what is the conditional probability that exactly 7 of the 10 flips land on heads?

4.50. Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are
(a) h, t, t (meaning that the first flip results in heads, the second in tails, and the third in tails);
(b) t, h, t.

4.51. The expected number of typographical errors on a page of a certain magazine is .2. What is the probability that the next page you read contains (a) 0 and (b) 2 or more typographical errors? Explain your reasoning!

4.52. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that there will be
(a) at least 2 such accidents in the next month;
(b) at most 1 accident in the next month?
Explain your reasoning!

4.53. Approximately 80,000 marriages took place in the state of New York last year. Estimate the probability that, for at least one of these couples,
(a) both partners were born on April 30;
(b) both partners celebrated their birthday on the same day of the year.
State your assumptions.

4.54. Suppose that the average number of cars abandoned weekly on a certain highway is 2.2. Approximate the probability that there will be
(a) no abandoned cars in the next week;
(b) at least 2 abandoned cars in the next week.

4.55. A certain typing agency employs 2 typists. The average number of errors per article is 3 when typed by the first typist and 4.2 when typed by the second. If your article is equally likely to be typed by either typist, approximate the probability that it will have no errors.

4.56. How many people are needed so that the probability that at least one of them has the same birthday as you is greater than $\frac{1}{2}$?