MATH 3355–1 FINAL EXAM Spring 2009

You may leave answers in terms of binomial coefficients, when applicable. ** SHOW ALL WORK **

(17) 1. A 2007 study showed that 17% of the citizens of Hawaii are smokers. Let X be the number of smokers in a randomly selected group of 35 Hawaii citizens.
(i) Treating X as a binomial random variable, find the probability that X is at least 2.
(ii) Now treat X as a Poisson random variable and again compute the probability that X is at least 2.
(iii) People are picked at random from this group of 35 citizens until a smoker is found. What is the probability that the first smoker found is the 4th person picked?
(iv) Approximating the binomial random variable X by a normal random variable, find the probability that $X = 7$. (Recall that the standard deviation of a binomial r.v. is $\sqrt{npq}$. A table for the function Φ appears on the last page of this exam.)

(9) 2. Let A and B be events in a sample space such that $P(A) = 0.4, P(B) = 0.7$, and $P(AB) = 0.2$. Determine $P(A \cup B), P(A|B)$, and $P(A^c B^c)$.

(12) 3. An experiment consists of the following: A number X is chosen at random from the set $\{1, 2, 3\}$, and then a fair die is thrown X times and we let Y equal the number of 5's or 6's that are rolled on these X throws. (So, for a given value of X, the random variable Y can take on the integers from 0 through X, and notice that $P(Y = 1|X = 1) = 1/3$.)
(i) Determine the joint distribution for X and Y (i.e., give a table).
(ii) Find the covariance of X and Y.

(9) 4. Three bowls B_1, B_2, and B_3 contain red and blue chips. The bowl B_1 contains two red and four blue chips; B_2 contains one red and two blue chips; and B_3 contains five red and four blue chips. In an experiment, one bowl is selected and then a chip is drawn from that bowl. However, the bowls are not selected with equal likelihood - B_1 is selected with probability 1/3, B_2 is selected with probability 1/6, and B_3 is selected with probability 1/2.
(i) Find the probability that a red chip is selected in this experiment.
(ii) Find the probability that a red chip is selected given that bowl B_3 was chosen.
(iii) The experiment is performed and a red chip is selected. What is the probability that the chip came from bowl B_3?

(8) 5. Two independent random variables X and Y are each uniformly distributed over the interval $[0, 1]$. Find the probability that $|X - Y|$ is less than 0.2.

(8) 6. Let X be a continuous random variable with probability density function

$$f_X(x) = \begin{cases} \frac{3}{16}\sqrt{x} & \text{if } 0 \le x \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

Find the cumulative distribution function of X.

(10) 7. Let the joint probability density function of continuous random variables X and Y be given by

$$f_{X,Y}(x,y) = \begin{cases} 10xy^2 & \text{if } 0 \le x \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E[X]$ and $E[Y]$.

(8) 8. A computer retail store has 12 personal computers in stock. A buyer wants to purchase 3 of them. Unknown to either the store or the buyer, 2 of the computers in stock have defective hard drives. Assume the computers are selected at random.
(i) What is the probability that exactly one of the 3 computers purchased will be defective?
(ii) What is the probability that at least one of the 3 computers purchased will be defective?

(9) 9. Let X be the discrete random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x < -3 \\ 3/8 & \text{if } -3 \le x < 0 \\ 1/2 & \text{if } 0 \le x < 3 \\ 3/4 & \text{if } 3 \le x < 4 \\ 1 & \text{if } x \ge 4 \end{cases}$$

Find $E(X)$ and $Var(X)$.

(10) 10. Scores on an IQ test given to all students are normally distributed with a mean of 100 and a standard deviation of 15.
(i) Students in the city of Red Stick are admitted to the gifted program if they score at least 130. What percentage of students qualify for this gifted program?
(ii) 36 students are selected at random and given an IQ test. What is the probability that the mean of their scores is above 105?