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There are 6 professors, 6 associate professors, 10 assistant professors and 12 instructors, so there are 34 people in all. We wish to form a committee of 6. There are $\binom{34}{6}$ ways to do this. We wish to count certain sets of possible committees. Let

A_1 = set of all possible committees that do not have any professors (the friendly, relaxed committees).

A_2 = set of all possible committees that exclude associate professors

A_3 = those that exclude assistant professors.

A_4 = those excluding instructors.

Then $A_1 \cup A_2 \cup A_3 \cup A_4$ = the set of all committees on which at least one of the ranks is excluded. By inclusion/exclusion.

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = |A_1| + |A_2| + |A_3| + |A_4|$$
$$- |A_1 A_2| - |A_1 A_3| - |A_1 A_4| - |A_2 A_3| - |A_2 A_4| - |A_3 A_4|$$
$$+ |A_1 A_2 A_3| + |A_1 A_2 A_4| + |A_1 A_3 A_4| + |A_2 A_3 A_4|$$
$$- |A_1 A_2 A_3 A_4|$$

Here $|S|$ denotes the number of elements in S.

$$= \binom{28}{6} + \binom{28}{6} + \binom{24}{6} + \binom{22}{6}$$
$$- \binom{22}{6} - \binom{18}{6} - \binom{16}{6} - \binom{18}{6} - \binom{16}{6} - \binom{12}{6}$$
$$+ \binom{12}{6} + \binom{10}{6} + \binom{6}{6} + \binom{6}{6}$$
$$- 0$$
$$= 835,144$$

$\binom{34}{6} = 1344904$. So $P(\text{excluding a rank}) = \frac{835,144}{1344904} \approx .620969\ldots$

So $P(\text{no rank excluded}) \approx 1 - .620969\ldots = .379031\ldots$