MATH 3355-1: Probability TEST 1 Name: Answers

1) An experiment consists of drawing 3 coins (all at once) from a purse that contains 3 pennies (dated 2001, 2002, and 2003), 2 nickels (dated 2004 and 2005) and a quarter (dated 2006).

a) [5] Describe the sample space

Label the coins: 1, 2, 3, 4, 5, 6.

The sample space consists of all 3-element subsets of $\{1, 2, 3, 4, 5, 6\}$

List the outcomes in the following events:

b) [5] getting more than 30 cents

$\{146, 246, 346, 156, 256, 356, 456\}$

c) [5] getting all three kinds of coin

$\{146, 246, 346, 156, 256, 356\}$

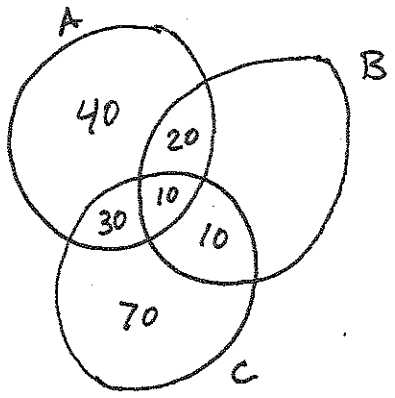
2) Event A contains 100 different outcomes <u>of which</u> 30 are in event B and 40 are in event C. There are 10 outcomes in the intersection of all three events.

a) [5] How many outcomes are in A but in neither B nor C?

40

b) [5] If in addition to the above, B and C have 20 outcomes in common (both in and out of A), and C contains 70 outcomes that are neither in A nor B, how many outcomes are in C?

120



3) [10] Find the probability of having at least one face card in a 5-card hand of poker. Provide an explanation for your work. (Hint.There are 12 face cards. There are 2,598,960 possible 5-card hands. How many have no face cards?)

$$P(\text{at least one face card}) = 1 - P(\text{no face card})$$

$$= 1 - \frac{\binom{40}{5}}{\binom{52}{5}}$$

$$= 1 - \frac{658,008}{2598,960} = 1 - .253181$$

$$= 0.746819$$

4) Dusty drank beer in **five** bars Friday night with his friends, leaving **four** $10-bills behind as tips. But he wasn't paying attention, and all the ways of distributing the bills among the bars can be assumed to have been equally likely.

a) [10] What's the probability that he left all four in one bar? (Provide an explanation for your work.)

There are 5^4 ways to distribute the bills.
There are 5 ways to leave all the bills in one bar.

Ans: $\frac{5}{5^4} = \frac{1}{125}$

b) [5] What's the probability that he left no more than one in each bar? (Provide an explanation for your work.)

$5 \cdot 4 \cdot 3 \cdot 2$ = # ways to choose a different bar for each bill.

Ans: $\frac{5 \cdot 4 \cdot 3 \cdot 2}{5^4} = \frac{24}{125} \approx \frac{1}{5}$

5) [10] Find the number of distinguishable permutations of the letters in "TENNESSEE TITANS". (Ignore the space).

T T T
E E E E
N N N
S S S
I
A

$$\frac{15!}{3!\,3!\,3!\,4!}$$

6) [10] What is the probability of getting a "four-flush" (*i.e.*, exactly four cards of the same suite) in a 5-card poker hand?

4 ways to choose a suite for the flush.
$\binom{13}{4}$ ways to choose 4 cards from that suite
39 ways to choose a card of another suite.

Ans: $\frac{4 \cdot \binom{13}{4} \cdot 39}{\binom{52}{5}} = 0,0429172\ldots$

7) [15] There are 5 white and 5 black balls in a box. Two are removed and replaced by black balls, then 5 are selected. What is the probability that all the selected balls are white?

The only outcome on the first draw that will leave 5 white balls in the box is a draw of two blacks.
The probability of 2 blacks on the first draw is $\binom{5}{2}/\binom{10}{2} = \frac{5\cdot 4}{10\cdot 9} = \frac{2}{9}$. The probability of drawing 5 whites, when there are 5 whites in the box is $1/\binom{10}{5} = 1/252$.

So, the probability of getting 5 whites on second draw is

$$\frac{2}{9}\cdot\frac{1}{252} = \frac{1}{(9)(126)} = \frac{1}{1134} = .000881834..$$

8) [15] 65% of the people in Dogpatch County are infected with cat scratch fever. 85% of those with cat scratch fever have a runny nose, and 23% of those without cat scratch fever have a runny nose. What's the probability that someone with a runny nose has cat scratch fever?

infected \ Runny nose	Yes	No	
yes	(.85)(.65)		.65
no	(.23)(.35)		.35
	(.85)(.65) + (.23)(.35)		

$$P(\text{infected} \mid \text{runny nose}) = \frac{P(\text{runny nose} \mid \text{infected})\,P(\text{infected})}{P(\text{runny nose})}$$

$$= \frac{(.85)(.65)}{(.85)(.65) + (.23)(.35)}$$

$$= \frac{.5525}{.5525 + .0805} = .872828...$$