3.5:35.

Extension a. An experiment has probability p of success, q of failure and $1-p-q$ of neither. If independent trials are repeated over and over until either success or failure is obtained, what is the probability of an ultimate success?

Solution. Let S be the event of success on the first trial, let F be the event of getting failure on first trial, let N be the event of getting neither on the first trial. Let E be the event of getting a success before a failure. Then

$$\begin{aligned} P(E) &= P(E|S)\cdot P(S) + P(E|F)\cdot P(F) + P(E|N)\cdot P(N) \\ &= 1\cdot p + 0\cdot q + P(E)\cdot(1-p-q) \\ &= p + P(E)\cdot(1-p-q). \end{aligned}$$

Thus,

$$0 = p - P(E)(p+q),$$

so

$$P(E) = \frac{p}{p+q}.$$

Extension b. In a lottery with a very large number of tickets, 1 in 100 tickets pay \$2, 1 in 1000 tickets pay \$10, 1 in 10000 tickets pay \$50. What is a ticket worth?

Solution. Let $X(\omega_i)$ be the payoff for the outcome of getting a ticket of type i, where $i=0$ if the ticket is a looser, $i=1$ if it wins \$2, $i=2$ if it wins \$10 and $i=3$ if it wins \$50. Thus, $X(\omega_0)=0$, $X(\omega_1)=2$, etc. Let f be the pmf.

$$\begin{aligned} E(X) &= X(\omega_0)f(\omega_0) + X(\omega_1)f(\omega_1) + X(\omega_2)f(\omega_2) + X(\omega_3)f(\omega_3) \\ &= (0)f(\omega_0) + (2)f(\omega_1) + (10)f(\omega_2) + (50)f(\omega_3) \\ &= (2)(1/100) + (10)(1/1000) + (50)(1/10000) \\ &= .035 = 3\tfrac{1}{2} \text{ cents} \end{aligned}$$

Part 2. Now suppose that on Mardi Gras, 1 in ten tickets wins two additional tickets in the same lottery. What is a ticket on Mardi Gras worth?

Solution. Let ω_4 be the event of getting the bonus Mardi Gas ticket. If $E(X)=e$, then $X(\omega_4)=2e$.

$$\begin{aligned} e = E(X) &= X(\omega_0)f(\omega_0) + X(\omega_1)f(\omega_1) + X(\omega_2)f(\omega_2) + X(\omega_3)f(\omega_3) + X(\omega_4)f(\omega_4) \\ &= (0)f(\omega_0) + (2)f(\omega_1) + (10)f(\omega_2) + (50)f(\omega_3) + (2e)f(\omega_4) \\ &= (2)(1/100) + (10)(1/1000) + (50)(1/10000) + (2e)(1/10) \end{aligned}$$

$$\begin{aligned} e &= .035 + e/5 \\ e &= .04375 = 4\tfrac{3}{8} \text{ cents} \end{aligned}$$