Lecture 9. Some Problems

Report on Homework due 02/08/2011

Problem 1. Suppose that 28 crayons, of which 4 are red, are divided randomly among Jack, Marty, Sharon and Martha (seven each). If Sharon has exactly one red crayon, what is the probability that Marty has the remaining 3? (Section 3.1, problem 17)

Solution by Yanshan Chen. If Sharon has received seven crayons, including exactly one red one, there are 21 crayons left to distribute, and there are three reds among them. Marty may receive any of the $\binom{21}{7}$ selections, and all are equally likely. To get all three reds in his hand of 7, Marty must receive all of them, as well as 4 of the 18 non-red crayons. There are $\binom{18}{4}$ ways. Thus, his chance of getting the three reds is:

$$\frac{\binom{18}{4}}{\binom{21}{7}} = \frac{7 \cdot 6 \cdot 5}{21 \cdot 20 \cdot 19} = \frac{1}{38}.$$

Problem 2. A child has run away. It is known that she must be in one of three locations, and the probability of being in region i is estimated to be α_i. (Note that $\alpha_1 + \alpha_2 + \alpha_3 = 1$.) It is also estimated that *if the child is in location i, then a search of that location will* <u>*fail*</u> *to find her* with probability β_i, $i = 1, 2, 3$. If regions 1 and 2 have been searched without success, what is the probability that the child is in region 3?

Solution by Alphonso Croeze. Define events A_3 and B as follows:

A_3 = the event that the child is in region 3;

B = the event that regions 1 and 2 have been searched unsuccessfully.

By assumption, $P(A_3) = \alpha_3$. $P(A_3 \cap B^c) = 0$, so $P(A_3 \cap B) = P(A_3) = \alpha_3$. B^c consists of two disjoint events: finding the the child in region 1, and finding the the child in region 2. $P(B^c)$, therefore, is the sum of the probabilities of those two events. The probability of finding the the child in region 1 is the probability of the child being in region i times the probability that region i is searched successfully. Therefore,

$$P(B^c) = \alpha_1(1 - \beta_1) + \alpha_2(1 - \beta_2),$$

and

$$P(B) = 1 - \alpha_1(1 - \beta_1) - \alpha_2(1 - \beta_2).$$

Thus,

$$P(A_3|B) = \frac{P(A_3 \cap B)}{P(B)} = \frac{\alpha_3}{1 - \alpha_1(1 - \beta_1) - \alpha_2(1 - \beta_2)}.$$

Solution by Jared Braud. Let A_i be the event that the child is in region i ($i = 1, 2, 3$). Let B be the event that searches of regions 1 and 2 are unsuccessful. Then

$$\begin{aligned} P(A_3|B) &= \frac{P(B|A_3) \cdot P(A_3)}{P(B)} \\ &= \frac{P(B|A_3) \cdot P(A_3)}{P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) \cdot P(A_3)} \\ &= \frac{\alpha_3}{\beta_1\alpha_1 + \beta_1\alpha_1 + \alpha_3}. \end{aligned}$$

Some additional examples.

x.y:z. means Chapter x, section y, problem z.

3.5:4. A die is rolled twice. Let A denote the event that the sum of the outcomes is odd, and B denote the event that it lands 2 of the first toss. Are A and B independent? Why or why not?

Extension. What if the die is rolled three times? Is the parity of the sum independent of the outcome on the first toss? Is the divisibility of the sum by 3 independent of the first toss?

3.5:8. If the odds in favor of an event are 3 to 1, what are the odds of the event occurring at least once in two independent trials?

Extension. If independent events have probabilities p and q, what are the odds of at least one of the events occurring?

3.5:10. What is the probability of getting *at least* one 6 in four throws of a fair die? What is the probability of getting at least one double 6 in 24 throws of a pair of dice?

Extension. What is the probability of getting *exactly* one 6 in four throws of a die? What is the probability of getting exactly one double 6 in 24 throws of a pair of dice?