EXERCISES 3.1

A

1. Suppose that 15% of the population of a country are unemployed women, and a total of 25% are unemployed. What percent of the unemployed are women?

2. Suppose that 41% of Americans have blood type A, and 4% have blood type AB. If in the blood of a randomly selected American soldier the A antigen is found, what is the probability that his blood type is A? The A antigen is found only in blood types A and AB.

3. In a technical college all students are required to take calculus and physics. Statistics show that 32% of the students of this college get A's in calculus, and 20% of them get A's in both calculus and physics. Gino, a randomly selected student of this college, has passed calculus with an A. What is the probability that he got an A in physics?

4. Suppose that two fair dice have been tossed and the total of their top faces is found to be divisible by 5. What is the probability that both of them have landed 5?

5. A bus arrives at a station every day at a random time between 1:00 P.M. and 1:30 P.M. A person arrives at this station at 1:00 and waits for the bus. If at 1:15 the bus has not yet arrived, what is the probability that the person will have to wait at least an additional 5 minutes?

6. Prove that $P(A \mid B) > P(A)$ if and only if $P(B \mid A) > P(B)$. In probability, if for two events A and B, $P(A \mid B) > P(A)$, we say that A and B are **positively correlated.** If $P(A \mid B) < P(A)$, A and B are said to be **negatively correlated.**

7. A spinner is mounted on a wheel of unit circumference (radius $1/2\pi$). Arcs A, B, and C of lengths $1/3$, $1/2$, and $1/6$, respectively, are marked off on the wheel's perimeter (see Figure 3.1). The spinner is flicked and we know that it is not pointing toward C. What is the probability that it points toward A?

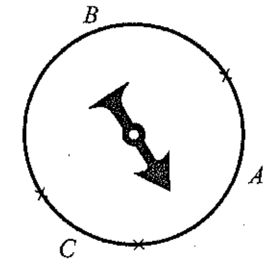

Figure 3.1 Spinner of Exercise 7.

8. In throwing two fair dice, what is the probability of a sum of 5 if they land on different numbers?

9. In a small lake, it is estimated that there are approximately 105 fish, of which 40 are trout and 65 are carp. A fisherman caught eight fish; what is the probability that exactly two of them are trout if we know that at least three of them are not?

10. From 100 cards numbered $00, 01, \ldots, 99$, one card is drawn. Suppose that α and β are the sum and the product, respectively, of the digits of the card selected. Calculate $P(\{\alpha = i \mid \beta = 0\})$, $i = 0, 1, 2, 3, \ldots, 18$.

11. From families with three children, a family is selected at random and found to have a boy. What is the probability that the boy has (a) an older brother and a younger sister; (b) an older brother; (c) a brother and a sister? Assume that in a three-child family all gender distributions have equal probabilities.

12. Show that if $P(A) = 1$, then $P(B \mid A) = P(B)$.

13. Prove that if $P(A) = a$ and $P(B) = b$, then $P(A \mid B) \geq (a + b - 1)/b$.

14. Prove Theorem 3.1.

15. Prove that if $P(E \mid F) \geq P(G \mid F)$ and $P(E \mid F^c) \geq P(G \mid F^c)$, then $P(E) \geq P(G)$.

16. The theaters of a town are showing seven comedies and nine dramas. Marlon has seen five of the movies. If the first three movies he has seen are dramas, what is the probability that the last two are comedies? Assume that Marlon chooses the shows at random and sees each movie at most once.

17. Suppose that 28 crayons, of which four are red, are divided randomly among Jack, Marty, Sharon, and Martha (seven each). If Sharon has exactly one red crayon, what is the probability that Marty has the remaining three?

B

18. A number is selected at random from the set $\{1, 2, \ldots, 10{,}000\}$ and is observed to be odd. What is the probability that it is (a) divisible by 3; (b) divisible by neither 3 nor 5?

19. A retired person chooses randomly one of the six parks of his town everyday and goes there for hiking. We are told that he was seen in one of these parks, Oregon Ridge, once during the last 10 days. What is the probability that during this period he has hiked in this park two or more times?

20. A big urn contains 1000 red chips, numbered 1 through 1000, and 1750 blue chips, numbered 1 through 1750. A chip is removed at random, and its number is found to be divisible by 3. What is the probability that its number is also divisible by 5?

21. There are three types of animals in a laboratory: 15 type I, 13 type II, and 12 type III. Animals of type I react to a particular stimulus in 5 seconds, animals of types II and III react to the same stimulus in 4.5 and 6.2 seconds, respectively. A psychologist selects 10 of these animals at random and finds that exactly four of them react to this stimulus in 6.2 seconds. What is the probability that at least two of them react to the same stimulus in 4.5 seconds?

22. In an international school, 60 students, of whom 15 are Korean, 20 are French, eight are Greek, and the rest are Chinese, are divided randomly into four classes of 15 each. If there are a total of eight French and six Korean students in classes A and B, what is the probability that class C has four of the remaining 12 French and three of the remaining nine Korean students?

23. From the set of all families with two children, *a family* is selected at random and is found to have a girl called Mary. We want to know the probability that both children of the family are girls. By Example 3.2, the probability should apparently be 1/3 because presumably knowing the name of the girl should not make a difference. However, if we ask a friend of that family whether Mary is the older or the younger child of the family, upon receiving the answer, we can conclude that the probability of the other child being a girl is 1/2. Therefore, there is no need to ask the friend, and we conclude already that the probability is 1/2! What is the flaw in this argument? Explain.

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