

REVIEW PROBLEMS

1. Two persons arrive at a train station, independently of each other, at random times between 1:00 P.M. and 1:30 P.M. What is the probability that one will arrive between 1:00 P.M. and 1:12 P.M., and the other between 1:17 P.M. and 1:30 P.M.?
2. A polygraph operator detects innocent suspects as being guilty 3% of the time. If during a crime investigation six innocent suspects are examined by the operator, what is the probability that at least one of them is detected as guilty?
3. In statistical surveys where individuals are selected randomly and are asked questions, experience has shown that only 48% of those under 25 years of age, 67% between 25 and 50, and 89% above 50 will respond. A social scientist is about to send a questionnaire to a group of randomly selected people. If 30% of the population are younger than 25 and 17% are older than 50, what percent will answer her questionnaire?
4. Diseases D_1 , D_2 , and D_3 cause symptom A with probabilities 0.5, 0.7, and 0.8, respectively. If 5% of a population have disease D_1 , 2% have disease D_2 , and 3.5% have disease D_3 , what percent of the population have symptom A? Assume that the only possible causes of symptom A are D_1 , D_2 , and D_3 and that no one carries more than one of these three diseases.
5. Professor Stern has three cars. The probability that on a given day car 1 is operative is 0.95, that car 2 is operative is 0.97, and that car 3 is operative is 0.85. If Professor Stern's cars operate independently, find the probability that on next Thanksgiving day (a) all three of his cars are operative; (b) at least one of his cars is operative; (c) at most two of his cars are operative; (d) none of his cars is operative.
6. A bus traveling from Baltimore to New York breaks down at a random location. If the bus was seen running at Wilmington, what is the probability that the breakdown occurred after passing through Philadelphia? The distances from New York, Philadelphia, and Wilmington to Baltimore are, respectively, 199, 96, and 67 miles.
7. Roads A, B, and C are the only escape routes from a state prison. Prison records show that, of the prisoners who tried to escape, 30% used road A, 50% used road B, and 20% used road C. These records also show that 80% of those who tried to escape via A, 75% of those who tried to escape via B, and 92% of those who tried to escape via C were captured. What is the probability that a prisoner who succeeded in escaping used road C?
8. From an ordinary deck of 52 cards, 10 cards are drawn at random. If exactly four of them are hearts, what is the probability of at least one spade being among them?

9. A fair die is thrown twice. If the second outcome is 6, what is the probability that the first one is 6 as well?
10. Suppose that 10 dice are thrown and we are told that among them at least one has landed 6. What is the probability that there are two or more sixes?
11. Urns I and II contain three pennies and four dimes, and two pennies and five dimes, respectively. One coin is selected at random from each urn. If exactly one of them is a dime, what is the probability that the coin selected from urn I is the dime?
12. An experiment consists of first tossing an unbiased coin and then rolling a fair die. If we perform this experiment successively, what is the probability of obtaining a heads on the coin before a 1 or 2 on the die?
13. Six fair dice are tossed independently. Find the probability that the number of 1's minus the number of 2's will be 3.
14. Urn I contains 25 white and 20 black balls. Urn II contains 15 white and 10 black balls. An urn is selected at random and one of its balls is drawn randomly and observed to be black and then returned to the same urn. If a second ball is drawn at random from this urn, what is the probability that it is black?
15. An urn contains nine red and one blue balls. A second urn contains one red and five blue balls. One ball is removed from each urn at random and without replacement, and all of the remaining balls are put into a third urn. What is the probability that a ball drawn randomly from the third urn is blue?
16. A fair coin is tossed. If the outcome is heads, a red hat is placed on Lorna's head. If it is tails, a blue hat is placed on her head. Lorna cannot see the hat. She is asked to guess the color of her hat. Is there a strategy that maximizes Lorna's chances of guessing correctly?
Hint: Suppose that Lorna chooses the color red with probability α and blue with probability $1 - \alpha$. Find the probability that she guesses correctly.
17. A child is lost at Epcot Center in Florida. The father of the child believes that the probability of his being lost in the east wing of the center is 0.75, and in the west wing 0.25. The security department sends three officers to the east wing and two to the west to look for the child. Suppose that an officer who is looking in the correct wing (east or west) finds the child, independently of the other officers, with probability 0.4. Find the probability that the child is found.
18. Solve the following problem, asked of Marilyn Vos Savant in the "Ask Marilyn" column of *Parade Magazine*, August 9, 1992.

Three of us couples are going to Lava Hot Springs next weekend. We're staying two nights, and we've rented two studios, because each holds a maximum of only four people. One couple will get their own studio on Friday, a different couple on Saturday, and one couple will be out of

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18. A number is selected at random from the set $\{1, 2, \dots, 10,000\}$ and is observed to be odd. What is the probability that it is (a) divisible by 3; (b) divisible by neither 3 nor 5?
19. A retired person chooses randomly one of the six parks of his town everyday and goes there for hiking. We are told that he was seen in one of these parks, Oregon Ridge, once during the last 10 days. What is the probability that during this period he has hiked in this park two or more times?
20. A big urn contains 1000 red chips, numbered 1 through 1000, and 1750 blue chips, numbered 1 through 1750. A chip is removed at random, and its number is found to be divisible by 3. What is the probability that its number is also divisible by 5?
21. There are three types of animals in a laboratory: 15 type I, 13 type II, and 12 type III. Animals of type I react to a particular stimulus in 5 seconds, animals of types II and III react to the same stimulus in 4.5 and 6.2 seconds, respectively. A psychologist selects 10 of these animals at random and finds that exactly four of them react to this stimulus in 6.2 seconds. What is the probability that at least two of them react to the same stimulus in 4.5 seconds?
22. In an international school, 60 students, of whom 15 are Korean, 20 are French, eight are Greek, and the rest are Chinese, are divided randomly into four classes of 15 each. If there are a total of eight French and six Korean students in classes A and B, what is the probability that class C has four of the remaining 12 French and three of the remaining nine Korean students?
23. From the set of all families with two children, a family is selected at random and is found to have a girl called Mary. We want to know the probability that both children of the family are girls. By Example 3.2, the probability should apparently be $1/3$ because presumably knowing the name of the girl should not make a difference. However, if we ask a friend of that family whether Mary is the older or the younger child of the family, upon receiving the answer, we can conclude that the probability of the other child being a girl is $1/2$. Therefore, there is no need to ask the friend, and we conclude already that the probability is $1/2$! What is the flaw in this argument? Explain. ■