Test 03/03/2011 Name_____

1. Consider the following experiments. In each case, provide the information requested.

(a) [3pts] A coin is flipped 3 times

An example of a typical outcome is: HHT

The total number of outcomes in the whole sample space is: 8

(b) [3pts] Three dice of different colors are rolled

An example of a typical outcome is: 621 (meaning 6 on first color, 2 on second, etc)

The total number of outcomes in the whole sample space is: 216

(c) [3pts] A card is drawn from a deck, and then without replacing it, another card is drawn

An example of a typical outcome is: (3H, 4D)

The total number of outcomes in the whole sample space is: 52·51

(d) [3pts] A card is drawn at random from a deck; it is replaced, and then another card is drawn

An example of a typical outcome *that is not in the sample space of* (c) is: (3H, 3H)

The total number of outcomes in the whole sample space is: 52·52.

2. A certain experiment has a sample space containing 100 different outcomes that are all equally probable. Event A contains 50 outcomes and event B contains 40 outcomes.

(a) [3pts] If A and B have 30 outcomes in common, what is the probability that *neither* A *nor* B occurs? 2/5

(b) [3pts] If A and B are *independent*, what is the probability that *both* A *and* B occurs? 1/5

(c) [3pts] If A and B are *mutually exclusive*, what is the probability that *either* A *or* B occurs? 9/10

3. Three letters (possibly with repeats) are selected at random from the alphabet and written in the order selected. What are the following probabilities? Your answer may be an expression such as $26 \cdot 25 \cdot 24$, and it may include symbols such as $\binom{5}{3}$. A *vowel* is one of the letters: *a, e, i, o, u*. A consonant is one the remaining 21 letters.

(a) [3pts] P(all the letters are vowels) $\left(\frac{5}{26}\right)^3$

(b) [3pts] P(the first and last letters are consonants and the middle one is a vowel)

$\left(\frac{21}{26}\right)\left(\frac{5}{26}\right)\left(\frac{21}{26}\right)$

(Continued...)

(c) [3pts] P(three *different* letters are written)

$$\left(\frac{25}{26}\right)\left(\frac{24}{26}\right)$$

(d) [3pts] P(one of the three letters appears exactly two times, e.g. "AAB", "ABA", etc.)

#outcomes in this event = 26 (choices for repeated letter) · 25 (choices for single letter) · 3 (ways to order the letters) = $3 \cdot 25 \cdot 26$. So,

$$P(\text{"}) = \frac{26 \cdot 25 \cdot 3}{26 \cdot 26 \cdot 26} = \frac{75}{26^2}$$

(e) [3pts] P(there is at least one vowel among the letters written)

$$1 - \left(\frac{21}{26}\right)^3$$

(f) [3pts] P(three *different* vowels are written, GIVEN THAT only vowels were chosen)

$$\frac{5 \cdot 4 \cdot 3}{5 \cdot 5 \cdot 5} = \frac{12}{25} = .48$$

(g) [3pts] P(the letters are in alphabetical order, GIVEN THAT all the letters are different)

$1/6$ (If three different letters have been chosen, there are 6 ways to order them, but only one is alphabetical).

4. These problems concern the binomial distribution

$$P(X = x \,|\, n, p) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n.$$

(a) [4pts] Explain the meaning of the above. What is X? What is x? What is n? What is p?

X is the number of successes in n independent trials if the prob. of success on any one is p. X is a random variable and x is one of its possible values.

(b) [4pts] If a fair coin is flipped 6 times, what is the probability that at most two heads are obtained?

$$\binom{6}{0}\left(\tfrac{1}{2}\right)^6 + \binom{6}{1}\left(\tfrac{1}{2}\right)^6 + \binom{6}{2}\left(\tfrac{1}{2}\right)^6 = \frac{11}{32}$$

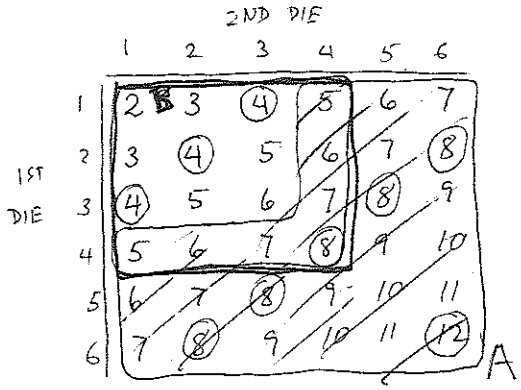
(c) [4pts] If a fair die is rolled 6 times, what is the probability that at most two sixes are obtained?

$$\binom{6}{0}\left(\tfrac{1}{6}\right)^0\left(\tfrac{5}{6}\right)^6 + \binom{6}{1}\left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right)^5 + \binom{6}{2}\left(\tfrac{1}{6}\right)^2\left(\tfrac{5}{6}\right)^4 = \frac{14 \cdot 5^5}{6^6}$$

5. Two dice are rolled. Let X be the total rolled (i.e., the total number of dots showing on the up faces). Let A be the event that at least one of the dice comes up on a number that is greater than or equal to 4. Let B be the event that both dice land on numbers less than or equal to 4. Let C be the event "X is divisible by 4."

(a) [5pts] Are A and B independent? Why or why not?

$P(A)P(B) = 1/3$
$P(A \cap B) = 7/36$
} not indep.



(b) [5pts] Are A and C independent? Why or why not?

$P(A)P(C) = 3/16$
$P(A \cap C) = 1/6$
} not indep.

○ = outcomes in C.

$P(A) = \frac{27}{36} = \frac{3}{4}$

$P(B) = \frac{16}{36} = \frac{4}{9}$

$P(C) = \frac{9}{36} = \frac{1}{4}$

$P(A \cap B) = 7/36$

$P(A \cap C) = 6/36 = 1/6$

$P(B \cap C) = 4/36 = 1/9$

(c) [5pts] Are B and C independent? Why or why not?

$P(B)P(C) = \frac{4}{9} \cdot \frac{1}{4} = 1/9$
$P(B \cap C) = 1/9$
} = indep.

6. [6pts] Two dice are rolled. Let W be the product of the numbers showing (e.g., if one die lands with 3 up and the other with 5, then $W = 15$). Find $E(W)$.

$$E(W) = \sum_{i=1}^{6}\sum_{j=1}^{6} i \cdot j \cdot \frac{1}{36} = \left(\sum_{i=1}^{6} i\right)\left(\sum_{j=1}^{6} j\right)\cdot\frac{1}{36} = \frac{21 \cdot 21}{36} =$$

$$= \frac{7 \cdot 7}{4} = \frac{49}{4} = 12.25$$

7. [10pts] The test for a certain disease is positive 98% of the time if given to a person with the disease, and it is positive 8% of the time if given to someone who does not have the disease. Only one in a hundred have the disease. If a random person tests positive, what it the probability she/he has the disease?

	Test: Pos	Test: Neg	
DISEASE: YES	.0098		.01
DISEASE: NO	.0792		.99
	.089		

BY (1)

These entries from (1) & given %s.

Sum column

$$P(\text{Disease} \mid \text{Postest}) = \frac{.0098}{.089} \approx \frac{1}{9}$$

$$\frac{(.98)(.01)}{(.98)(.01)+(.08)(.99)} = \frac{.0098}{.089}$$

Another way:

$$P(\text{Disease} \mid \text{Postest}) = \frac{P(\text{Pos.test} \mid \text{Disease})\,P(\text{Disease})}{P(\text{Postest} \mid \text{Disease})\,P(\text{Disease}) + P(\text{Postest} \mid \text{NoDis})\,P(\text{NoDis})}$$

8. An experiment has three possible outcomes with probabilities a, b and c, respectively. (Thus $a+b+c=1$.) The experiment is performed three times, with each trial independent of the others.

(a) [5pts] What is the probability that each of the outcomes occurs once?

Let α, β, γ be the outcomes (for each trial). If there are 3 trials, the outcomes are:

(α,α,α), (α,α,β), ... etc.

The event we are interested in is

$\{(\alpha,\beta,\gamma), (\alpha,\gamma,\beta), (\beta,\alpha,\gamma), (\beta,\gamma,\alpha), (\gamma,\alpha,\beta), (\gamma,\beta,\alpha)\}$

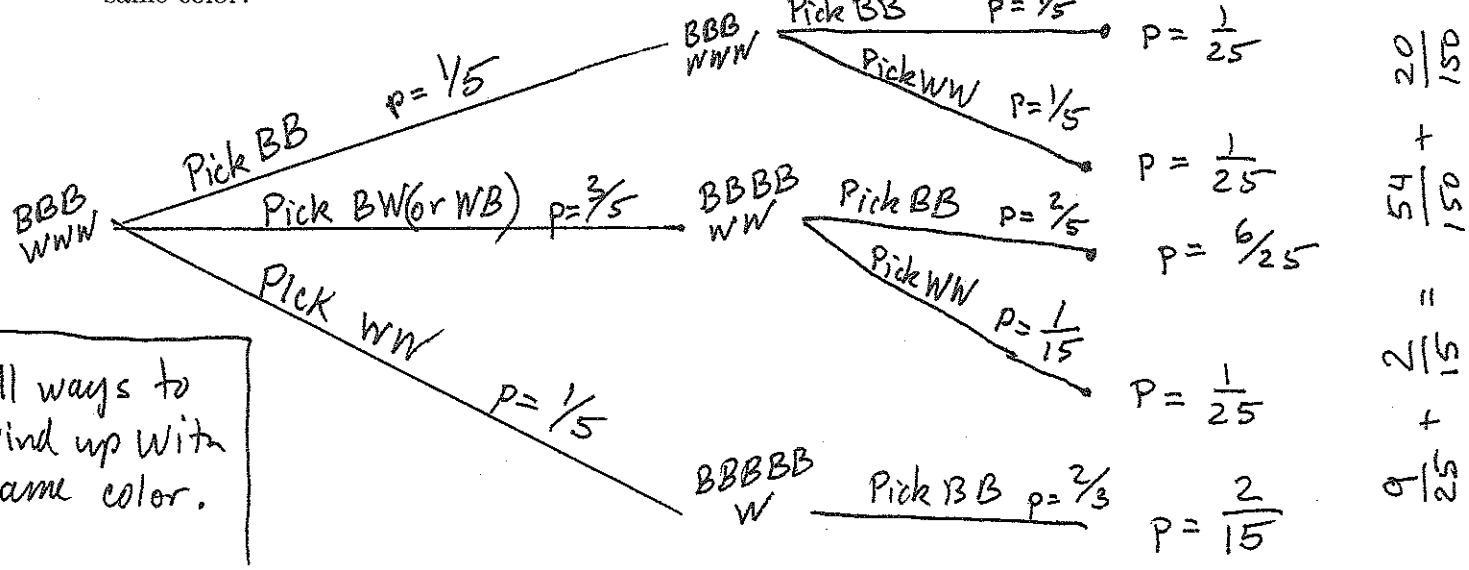
Each outcome in this event has prob. abc. So $P(\text{event}) = 6abc$.

(b) [5pts] What is the probability that all three outcomes are the same?

$$E = \{(\alpha,\alpha,\alpha), (\beta,\beta,\beta), (\gamma,\gamma,\gamma)\}$$

$$P(E) = a^3 + b^3 + c^3.$$

9. [5pts] There are 3 white and 3 black balls in a box. Two are taken at random, removed and replaced by black balls, then 2 are selected. What is the probability that the selected balls are the same color?



All ways to wind up with same color.

$$\frac{74}{150} \approx .49333 = \frac{20}{150} + \frac{54}{150} + \frac{2}{15} + \frac{9}{25}$$