

The Exponential Distribution

A large population declines through the deaths of individuals in such a manner that at the end of each unit of time the population remaining is a fixed fraction the population at the beginning of that unit. Each individual has the same chance of death in any time interval, and the death of any individual has no influence on the deaths of any others.

Let $H(t)$ be the population at time t , and let $G(t) := H(t)/H(0)$, so $G(t)$ is the population at time t measured in terms of the population at time 0. You learned in high school that:

$$G(t) = e^{-\lambda t}, \quad \text{where } \lambda := -\ln(G(1)).$$

1. What is the probability that a random individual in this population dies before time t ?
2. Show that the function you just wrote is a cdf on $[0, \infty)$.
3. Find the corresponding pdf.
4. Describe the meaning of the corresponding random variable.
5. What is the life expectancy of a random individual, in terms of λ ?
6. What is the variance of the distribution whose pdf you wrote in answer to question 3?